

# Behavioral Power Amplifier Model considering Memory Effects dedicated to radar system simulation

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**Abstract** — In radar systems, where pulsed RF signals are used, one of the main concern is the spurious emission. Such spurious are emissions of frequencies outside the bandwidth of interest. The spurious level must be kept under a Aaaaa level to be compliant with the specifications. In order to check all these specifications, system level simulation can be used, but accuracy and reliability of the simulation results will depend on the circuit model reliability, especially for the Power Amplifier (PA) which is a critical element. Such model must take into account the different memory effects. This paper proposes a complete and practical methodology to extract a Behavioral PA model dedicated to radar applications. A specific attention is paid on the coupling effects between short and long term memory dynamics.

**Index Terms** — Behavioral model, SSA, Memory effects, Non linear memory, Complex Envelope, radar, short and long pulses.

## I. INTRODUCTION

In order to check if a radar system is compliant with the specifications, and to avoid costly test procedure of a complete radar system which is made of hundreds/thousands of sub-systems, the current trend is to replace measurement procedures by an equivalent system simulation.

As an example, within a system such as an Active Electronically Scanned Array (AESA), each antenna is driven by a front/end circuit. Thus, the simulation of the active antenna, loaded by a great amount of front/end circuits, is not realistic if the co-simulation is linked to a circuit level simulation, because of speed and convergence matters. Thus, to allow first-pass system design success, accurate behavioral models of RF front-end circuits is nowadays a hot topic.

When dealing with pulsed signals, an efficient “black-box” model must be able to determine the non-linear dispersive effects accurately without requiring massive amounts of computational resources. If accurate, such models must take into account short term and long term memory effects.

The short-term memory effects are mainly driven by the RF carrier frequency, according to the characteristic of the matching networks within the PA.

The long-term memory effects, by definition occur at lower frequencies, and can strongly impact the PA's response over time. They are mainly caused by inter-modulation low frequency products, which can trigger some kind of bias modulation, dynamic heating or cooling caused by the envelop of the modulated signal, or any kind of parasitic effects such as trapping effects in Field Effect Transistors.

This paper describes a behavioral modeling methodology which has been used to extract a PA model for pulse radar applications, and which take into account all these complex phenomena.

## II. BACKGROUND

A first reasonable approach of PA modeling is to focus on the memory-effect impact within the RF bandwidth. Parallel and/or series associations of linear filters and memory-less nonlinearities were reported in [1], [2], [3], [4]. Unfortunately, this approach does not take into account the long term memory effects.

Alternatively, the PA behavioral models based on direct Volterra series expansion and digital filter representation adopted in [5], and [6] may lead to complex model architectures and intricate optimization procedures.

The combination of first-order modified Volterra series expansions reported in [7][8][9] enables a relatively straightforward extraction methodology with a good prediction of short- and long-term memory effects. Recently, the Poly-Harmonic Distortion (PHD) nonlinear behavioral model [10]-[11]-[12]-[13], has been proposed as a natural extension of S-parameters to nonlinear operating conditions. This model is based on linearization of the nonlinear response around a large tone signal which drives the PA into nonlinear conditions. The nonlinear behavior is then caught using a probe tone by measuring the scattering responses to the small harmonic signals applied in addition to the large tone. A main drawback with the PHD model formalism comes from the fact that these models are memory-less by nature, although a method has been proposed in [14] to include short-term memory effects using a Volterra series approach, completed by a similar method proposed in [15] to add long-term memory effects. Taking into account such short term or long memory effects is truly an improvement of the initial PHD modeling approach, but having both effects simultaneously and their mutual coupling is quite important, especially when the envelop modulation occurs in combination with frequency or phase modulation, such as in radar applications.

## III. MODEL TOPOLOGY

Here is presented the system-level characterization and the modeling methodology, dedicated to microwave amplifiers

designed for pulsed radar applications. Consider a PA circuit which suffers from memory effects, with the incident and scattered waves  $a_i(t)$ ,  $b_i(t)$  corresponding to the incident and reflected power waves on port numbers  $i=1,2$ .

The output wave  $b_i(t)$  can be expressed as a sum of fundamental and harmonic modulated tones (1):

$$b_i(t) = \text{Re} \left\{ \sum_{k=0}^N \tilde{b}_{ik}(t) e^{j2\pi k f_0 t} \right\} \quad (1)$$

Assuming that the main nonlinearity is driven by the incident power wave  $a_{11}(t)$  at the fundamental frequency, harmonic superposition can be applied and resulting in a relationship governing two-port nonlinear systems:

$$\tilde{b}_{ik}^{ST}(t) = \sum_{jl} S_{ik,jl}(|\tilde{a}_{11}(t)|) P^{k-l} \tilde{a}_{jl}(t) + \sum_{jl} T_{ik,jl}(|\tilde{a}_{11}(t)|) P^{k+l} \tilde{a}_{jl}^*(t) \quad (3)$$

with  $P = e^{j\omega_{m1} t}$

However, the terms  $S_{ik,jl}$  and  $T_{ik,jl}$  described by the incident wave  $a_{11}$  under steady-state conditions do not handle the memory effects. To address such effects, as shown in [14], dynamic Volterra-series expansion of the  $S_{ik,jl}$  and  $T_{ik,jl}$  terms can be used, as illustrated by equation (4).

$$\tilde{b}_{ik}(t) = \sum_{jl} \left\{ S_{ik,jl}(|\tilde{a}_{11}(t)|) P^{k-l} \tilde{a}_{jl}(t) + \frac{1}{2\pi} \int_{-\infty}^{+\infty} S_{ik,jl}^{ST}(|\tilde{a}_{11}(t)|, \Omega) P^{k-l} \tilde{a}_{jl}(\Omega) e^{j\Omega t} d\Omega \right\} + \sum_{jl \neq 1,1} \left\{ T_{ik,jl}(|\tilde{a}_{11}(t)|) P^{k+l} \tilde{a}_{jl}^*(t) + \frac{1}{2\pi} \int_{-\infty}^{+\infty} T_{ik,jl}^{ST}(|\tilde{a}_{11}(t)|, -\Omega) P^{k+l} \tilde{a}_{jl}^*(\Omega) e^{-j\Omega t} d\Omega \right\} \quad (4)$$

The model extension, proposed here, is based on a feedback loop principle. This feedback loop consists of two basic nonlinear dynamics with widely separated time constants. A feed-forward approach will represent the short term memory effects due to HF non linearity and RF Matching circuit influence (Fig 1). The weaker nonlinear dynamic is the long-term response; when a variable envelope signal cross the feed-forward block, a portion of the output signal is fed back to the input. The resulting principle is represented by the feedback loop sketched in Fig 1, reduced here to the fundamental frequency in order to represent the overriding influence of  $\tilde{a}_{11}(t)$ .

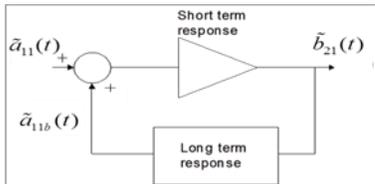


Fig. 1. Feedback loop principle

From the closed loop equation derived from Fig 1 and assuming the fact that the contribution is reasonably small; the final expression (5) presented in [16] is a good approximation of the SSPA's response.

$$\tilde{b}_{ik}(t) = \tilde{b}_{ik}^{ST}(t) \tilde{b}_{ik}^{FB}(t)$$

$$\tilde{b}_{ik}^{ST}(t) = \sum_{jl} \left\{ \frac{1}{2\pi} \int_{-\infty}^{+\infty} S_{ik,jl}^{ST}(|\tilde{a}_{11}(t)|, \Omega) \tilde{a}_{jl}(\Omega) e^{j\Omega t} d\Omega + \frac{1}{2\pi} \int_{-\infty}^{+\infty} T_{ik,jl}^{ST}(|\tilde{a}_{11}(t)|, -\Omega) \tilde{a}_{jl}^*(\Omega) \frac{\tilde{a}_{jl}(t)}{\tilde{a}_{jl}^*(t)} e^{-j\Omega t} d\Omega \right\}$$

$$\tilde{b}_{ik}^{FB}(t) = 1 + \int_0^{\infty} S_{ik,11}^{FB}(|\tilde{a}_{11}(t-\tau)|, \tau) |\tilde{a}_{11}(t-\tau)| d\tau \quad (5)$$

In (5)  $\tilde{b}_{ik}^{FB}(t)$  traduces the effects of the feedback loop on the output  $\tilde{b}_{ik}^{ST}(t)$  which implicates only short terms dynamics. The assumption of equation (5) states that  $\tilde{b}_{ik}^{FB}(t)$  term is mainly driven by the input signal at the fundamental frequency. In this case, long term memory effects are taken into account. Nevertheless the first assumption of equation (5) suffers from a lack of generality when the bandwidth is about several hundred of MHz. In [16], because the extraction of  $S_{ik,11}^{FB}$  is made at the center frequency of the bandwidth, it restricts the use of equation (5) to a narrowband modulation around a fixed RF carrier, or to a wideband modulation if the coupling effects between high frequency and low frequency memory are independent of the RF carrier. Because this last hypothesis is not guarantee in real conditions, this paper proposes to extend the model validity to operating conditions where short and long memory coupling effects are taken into account, though a dynamic parameterization of the kernel  $S_{ik,11}^{FB}$ . Using the instantaneous frequency parameter, a new model formulation is proposed in (6):

$$\tilde{b}_{ik}^{FB}(t) = 1 + \int_0^{\infty} S_{ik,11}^{FB}(|\tilde{a}_{11}(t-\tau)|, \dot{\varphi}(t), \tau) |\tilde{a}_{11}(t-\tau)| d\tau \quad (6)$$

with  $\dot{\varphi}(t) = \frac{\partial \varphi(t)}{\partial t}$

From a mathematical point of view, equation (6) is a decomposition of an arbitrary function truncated to the first order. This one is physically a linear sum of elementary nonlinear impulse responses. This formalism is not a linear contribution of the signal variation; as for dynamic Volterra series. This one is more suited for long term memory description.

#### IV. MODEL EXTRACTION

This section describes the measurement setup developed in order to extract the model's kernels of a Power Amplifier operating in X-band:

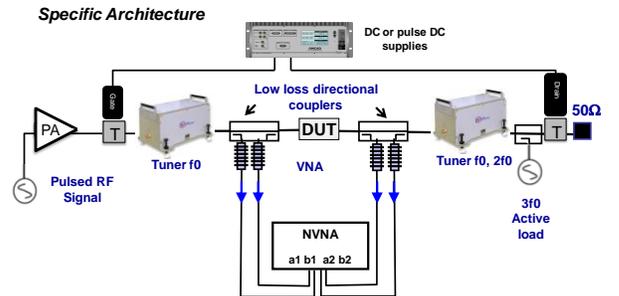


Fig. 2. Setup for Model Extraction

This setup is driven by IVCAD software; the NVNA used enables point-in-pulse measurements, and RF time-domain waveform reconstruction. The pulsed power waves are then measured thanks to low loss couplers. The harmonic load impedances are controlled through a hybrid tuning station. A key point for the extraction of kernels of (5) and (6) is to run the measurements within a measurement window which corresponds to the steady state part of the RF pulse, for both input and output ports of the DUT, whatever the level of the RF power.

#### A. Extraction of the short term kernels

The samples used are only located within the steady state measurement window. Thus, the measurement data are equivalent to a continuous wave characterization. This allows performing the extraction of the short-time kernels, as described in [14]. In a first step, the short-term kernel  $S_{21,11}^{ST}$  is unambiguously identified by driving the amplifier with a single tone signal, where the component  $\tilde{b}_{ik}^{LT}(t)=1$ ,  $|\tilde{a}_{11}(t)|$  are time independent. The extraction method sets the a-waves to zero, as done for S-parameter measurements, thus only a single pair of S&T terms will remain. For F0 & 2F0 harmonic measurements, b2 wave is considered, and a21, a12 and a22 are set to zero in order to extract the S21,11 and T21,11 terms. Harmonic load pull can be done to control impedances at both fundamentals and harmonic frequencies, thereby setting the a-waves to zero in accordance with the reference impedance. Thus, for different power levels  $|\tilde{a}_{11}|$  and frequencies  $\Omega$  the sets of each kernels  $S_{ik,jl}^{ST}(|\tilde{a}_{11}|, \Omega)$  and  $T_{ik,jl}^{ST}(|\tilde{a}_{11}|, -\Omega)$  are obtained. The power sweeps are roughly made of 30 points, and the gain compression must reach a value of 3-4 dBc. About 15 frequencies  $\Omega$  are used for a proper identification described in [14].

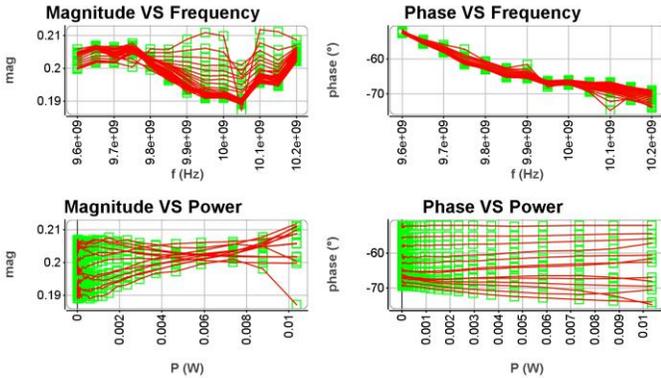


Fig. 3. IVCAD Identification of  $S_{11,11}^{ST}$  kernels

The dots correspond to measured data and the lines are analytical functions with poles/residues identification as described in [14].

#### B. Extraction of the feedback kernels

The feedback kernels  $S_{ik,11}^{FB}(|\tilde{a}_{11}(t-\sigma)|, \dot{\varphi}(t), \sigma)$  are determined by driving the amplifier with a unit-step envelope for different carrier frequencies, within the amplifier's RF bandwidth. Dividing the amplifier output response by the short-term response provides a signal  $b_{ik,jl}^2(t)$  that can be used to extract the impulse response  $S_{ik,11}^{FB}$ . For a proper representation of the non-linear impulse response, about 20 time samples are needed. An over-sampling FFT is used in order to obtain the HL-LF coupling kernels  $S_{ik,11}^{FB}(|\tilde{a}_{11}|, \dot{\varphi}(t), \Omega)$ .

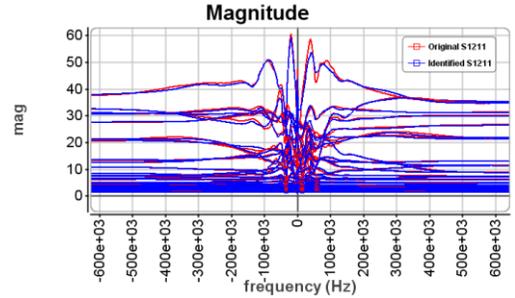


Fig. 4. IVCAD Identification of  $S_{12,11}^{FB}$  centered at 9.9 GHz.

The variations of the  $S_{12,11}^{FB}(|\tilde{a}_{11}|, \Omega)$  kernels highlight low frequency resonances mainly included in the 400KHz bandwidth around the carrier which are due to thermal effects.

### V. SIMULATION RESULTS AND MODEL VALIDATION

This model can be used in any commercial software which runs envelope or Harmonic-Balance simulations.

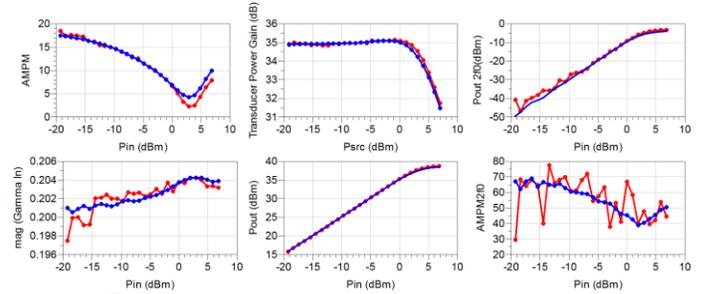


Fig. 5. HB simulation, model (blue)/measurement (red)

Fig 5 shows the performances of the model in comparison with the measurements looking at power gain, f0 and 2f0 Pout, Input impedance and AM/PM for a fundamental frequency of 9.8 GHz. Nevertheless, a degradation of the model accuracy can be observed for a gamma load higher than 0.4.

This model has been tested for different operating conditions, using two radar scenarios, with short and long RF pulse widths. Short pulse width is about 1μs for a period of 10μs, while long pulse width is about 50μs with a period of 50ms.

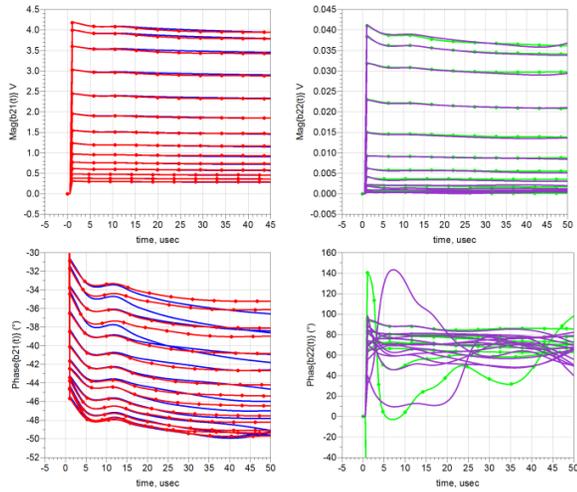


Fig. 2. Time domain envelope  $b_{21}(t)$  and  $b_{22}(t)$  in pulsed conditions

The model (dots) provides a very good accuracy for the simulation of the nonlinear long-term time constants (lines). These comparisons are made under low mismatched condition at  $f_0$  ( $G_{Load} = 0.25.e^{-j.165^\circ}$ ).

Another key point is to check the spectral shape prediction for pulsed RF signals used in radar applications (pulse width 1  $\mu$ s and period 10  $\mu$ s). The model (dots) and the measurements (lines) are very closed.

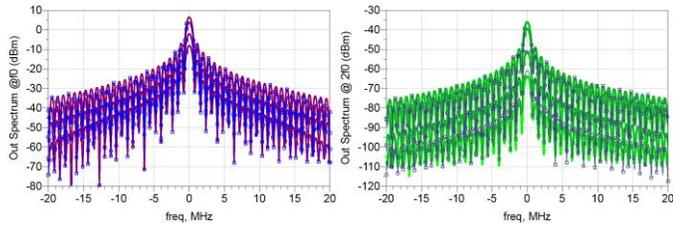


Fig. 3. Spectral analysis on 40 MHz BW centered around  $F_0$  (GHz)

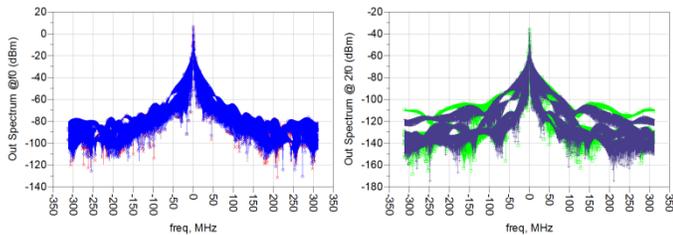


Fig. 4. Spectral analysis on 600 MHz BW centered around  $F_0$  (GHz)

The results provided by the proposed modeling approach are fairly good according to the representation of the spectral shape over a large RF bandwidth

## VI. CONCLUSION

An efficient new dynamic mapping technique based on Volterra expansion has been proposed. It has been successfully proven that this model can accurately reproduce the nonlinear envelope's distortions under low impedance's mismatch conditions. The proposed model extraction is

straightforward using data extracted from both circuit-level simulation and time-domain RF envelope measurements. The extraction principle does not require a parameter optimization process. The model can be extended in order to predict the DC power consumption, which is a critical parameter of power amplifiers and system link budgets!

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