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# BLACK BOX MODELLING OF HARD NONLINEAR BEHAVIOR IN THE FREQUENCY DOMAIN

J. Verspecht*, D. Schreurs**, A. Barel***, B. Nauwelaers**

* Hewlett-Packard NMDG, VUB-ELEC, Pleinlaan 2, 1050 Brussels, Belgium,
tel. 32-2-629.2886, fax 32-2-629.2850, email janv@james.belgium.hp.com

** K. U. Leuven, ESAT-TELEMIC, Kardinaal Mercierlaan 94, B-3001 Heverlee, Belgium

*** Vrije Universiteit Brussel, ELEC, Pleinlaan 2, 1050 Brussels, Belgium

***Abstract* — A black box model is proposed to describe nonlinear devices in the frequency domain. The approach is based upon the use of describing functions and allows a better description of hard nonlinearities than an approach based upon the Volterra theory. Simulations and experiments are described illustrating the mathematical theory.**

## I. Introduction

Describing and measuring nonlinear behavior of electrical components is important for many applications. The input signals are often very well approximated by a sum of sinewaves, a frequency domain approach is then preferred.

Two kinds of black box models are used in practice for the description of nonlinear systems in the frequency domain: Volterra series (cf. the VIOMAP model [1]) and describing functions [2]. The Volterra series approach can handle more than one spectral component present at the input, but the approach only works for weakly nonlinear behavior. The approach of the describing functions does not have this restriction. The idea is to write a spectral output component as a general function of the spectral input components. In practice this technique is only used when there is only one spectral component present at the input. In this text the describing function approach for several spectral input components is developed in order to model hard nonlinear behavior in the frequency domain [3].

## II. Theory

### A. Expressing time invariance

Consider a nonlinear device with at his input several spectral components. For simplicity it is assumed that the frequencies of the spectral components are commensurate, such that each frequency can be assigned an integer index, which equals the frequency divided by the fundamental frequency. Excluding chaotic and subharmonic behavior, the output spectrum will consist out of spectral components with frequencies which are integer multiples of the fundamental frequency. This implies that an integer index can be assigned to each spectral output component.

Consider N spectral input components where the $i^{th}$ component is represented by the complex number $I_{\alpha_i}$, with $\alpha_i$ equal to the frequency index. If one denotes the output component with frequency index k by $O_k$, one can write:

$$O_k = F_k(I_{\alpha_1}, I_{\alpha_2}, ..., I_{\alpha_N}) . \qquad (1)$$

$F_k(...)$ represents the describing function which maps the N complex numbers representing the input signal into the $k^{th}$ spectral component of the output signal. It can be shown that expressing that applying a delay at the input has to correspond to the same delay at the output (time invariance) results in:

$$O_k = (V_N)^k G_k(A_1, ..., A_N, V_1, ..., V_{N-1})$$

$$, \qquad (2)$$

where $G_k$ represents an arbitrary function (called describing function) and where $(A_1, ..., A_N, V_1, ..., V_N)$ are found by applying the following transformation on $(I_{\alpha_1}, I_{\alpha_2}, ..., I_{\alpha_N})$ :

$$A_i = \left|I_{\alpha_i}\right|, \qquad (3)$$

$$V_i = P_1^{s_{1i}} ... P_N^{s_{Ni}}, \qquad (4)$$

with $P_i = e^{j\varphi(I_{\alpha_i})}$, and with $(s_{1i}, ..., s_{Ni})$ for i ranging from 1 to N equal to N-1 linearly independent integer solutions of the equation:

$$\alpha_1 s_{1i} + ... + \alpha_N s_{Ni} = 0, \qquad (5)$$

and with $(s_{1N}, ..., s_{NN})$ an integer solution of the equation:

$$\alpha_1 s_{1N} + ... + \alpha_N s_{NN} = 1. \qquad (6)$$

### *B. Correspondence with the Volterra series*

It can be shown that the approach based upon Volterra series (VIOMAP) is a subset of the describing function approach (2), where the class of functions $G_k$ is constrained to a limited set of polynomials.

## III. Simulations

### *A. Introduction*

Simulations illustrate the above. A static nonlinearity is chosen described by:

$$y = \tanh(x). \qquad (7)$$

A simulation of an harmonic distortion analysis is performed on this system (peak amplitude of the input cosinusoidal wave swept from 0 to 10). For each amplitude the value of several harmonics is calculated. A plot of the fundamental and some harmonics is shown in Fig. 1.

### *B. Comparing two parametric models*

The performance is compared of two kinds of parametric models to be fitted on the data: a

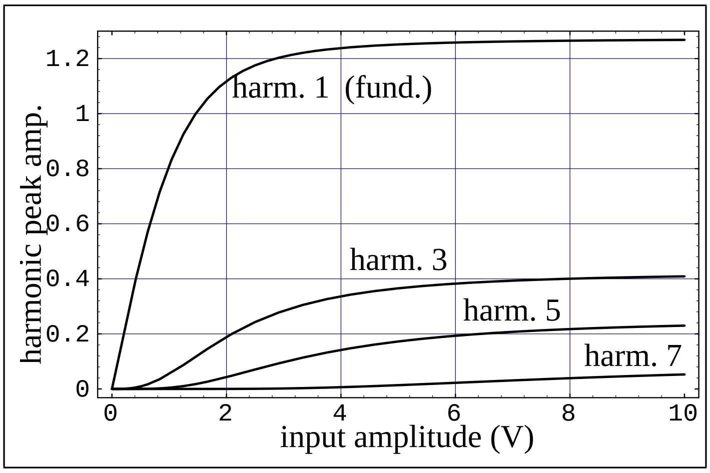


**Fig. 1 Harmonic distortion (harm. 1, 3, 5, 7).**

VIOMAP model and a rational model, based upon describing functions. The models are defined as (N equals the number of model parameters):

the VIOMAP polynomial (VIO):

$$O_k = (P_1)^k \left( \sum_{i=0}^{N} K_i A_1^{2i+k} \right), \qquad (8)$$

the rational model (RAT):

$$O_k = (P_1)^k \left( \sum_{i=0}^{N} K_i A_1^i \right) \left( \frac{A_1^k}{1 + A_1^k} \right). \qquad (9)$$

The parameters $K_i$ of the models (for different k and N) are estimated by means of a least-square fit to 200 uniformly sampled points of the harmonic distortion analysis characteristics. Note that the rational model is chosen such that it behaves like a $k^{th}$ order nonlinearity for $A_1$ much smaller than 1, and as general polynomial for $A_1$ much greater than 1. This way the model behaves like a classical Volterra approach for small input amplitudes, but has more flexibility at larger amplitudes, where the Volterra approach fails. Two measures are used to compare the models: the root-mean-square error ($e_{rms}$) and the maximum error ($e_{max}$). Values are given in Table 1 and Table 1.

The simulation reveals that, for the same number of parameters, RAT outperforms VIO.

**Table 1 Quality of the models for the fundamental (k=1).**

| N | $e_{rms}$ VIO | $e_{rms}$ RAT | $e_{max}$ VIO | $e_{max}$ RAT |
|---|---|---|---|---|
| 1 | -11 dB | -27 dB | -3 dB | -18 dB |
| 3 | -19 dB | -35 dB | -9 dB | -20 dB |
| 5 | -26 dB | -41 dB | -15dB | -27 dB |

**Table 2 Quality of the models for the 7th harmonic (k=7).**

| N | $e_{rms}$ VIO | $e_{rms}$ RAT | $e_{max}$ VIO | $e_{max}$ RAT |
|---|---|---|---|---|
| 1 | -28 dB | -31 dB | -19 dB | -22 dB |
| 5 | -39dB | -65 dB | -29 dB | -52 dB |
| 9 | -29 dB | -91 dB | -20 dB | -78 dB |

## IV. Experiments

### *A. Introduction*

A resistive mixer [4] experiment is performed on a broadband field effect transistor using a "nonlinear network" analyzer [5][6]. The local oscillator (lo) signal (3 GHz) is a voltage wave arriving at the gate of the transistor, while the rf-signal (4 GHz) is a voltage wave send towards the drain of the transistor (no dc-biasing present). The scattered voltage wave at the drain contains a lot of intermodulation products, with the two most important ones being the intermodulation products at 1 GHz and 7 GHz. To illustrate the theory, only the mixing product at 1 GHz is considered.

### *B. The measurements*

A set of two-tone measurements is performed on the "resistive mixer": the local oscillator as well as the rf-signal peak amplitude are swept from about 100 mV to 800 mV. There are 16 different values for the local oscillator power and 21 for the rf-signal amplitude, logarithmically distributed over the range. At each power setting the phase relationship between the two components is randomized 30 times. This results in a set of 21 times 16 times 30 equals 10080 measurements.

### *C. Modelling*

Two models are fit on the measured data: a VIOMAP and a rational model. The polynomial degree of the VIOMAP model is noted D. The rational model has the following form:

$$O_1 = P_4 P_3^* \left( \sum_{i=0}^{N} \sum_{j=0}^{N} \sum_{m=-M}^{M} K_{ijm} A_3^i A_4^j P_4^{3m} P_3^{*4m} \right) \left( \frac{A_3 A_4}{0.04 + A_3 A_4} \right), \quad (10)$$

where $K_{ijm}$ represents the model parameters. Note that the rational model is chosen such that it behaves like a classical Volterra for values of the product $A_3A_4$ smaller than 0.04, which corresponds roughly to the limit of "weakly nonlinear behavior". Next the parameters are extracted for several model orders, as well for the rational approach as for the VIOMAP. The rms errors are given in Table 3 ("NofP" denotes the number of parameters). The rational model achieves -47 dBV with 27 parameters, where the VIOMAP needs 62 parameters for reaching the same level. The above suggest the rational model to be better. More evidence is found looking at the maximum value of the error. For the rational model $e_{MAX}$ is at a level of -37 dBV, for the VIOMAP this value equals -31 dBV.

**Table 3 Root-mean-square errors of the different models.**

| Model | Degree | NofP | $e_{rms}$ (dBV) |
|---|---|---|---|
| VIO | D = 7 | 9 | -39 |
| VIO | D = 9 | 17 | -42 |
| VIO | D = 11 | 28 | -45 |
| VIO | D = 15 | 62 | -47 |
| RAT | N = 2, M = 1 | 27 | -47 |
| RAT | N = 3, M = 1 | 48 | -48 |

### D. Interpretation of the rational model

One can then use the rational model (N=2, M=1) in order to know the behavior of the described resistive mixer. Fig. 2 illustrates that,

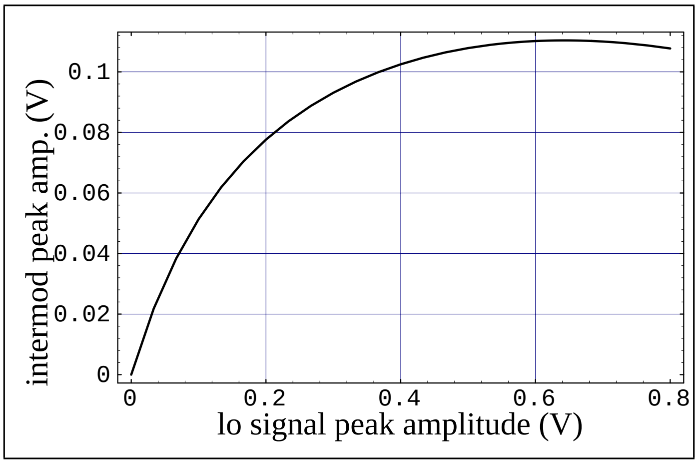


**Fig. 2 Intermod amp. vs lo amp. (rf amp. 0.2 V)**

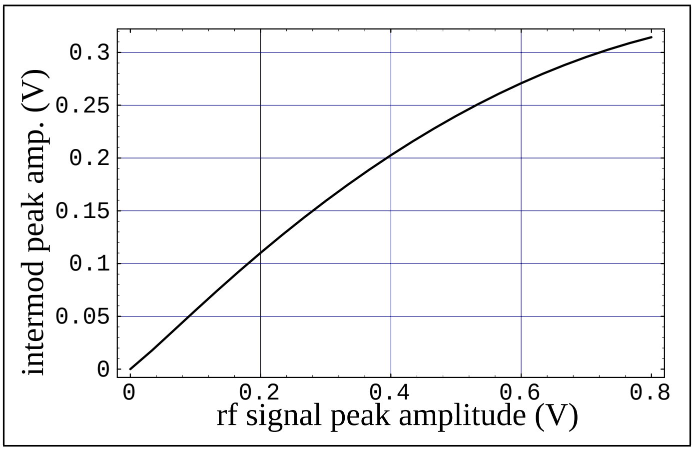


**Fig. 3 Intermod amp. vs rf amp. (lo amp. 0.6 V)**

for a small and constant rf amplitude, the intermod amplitude is no longer a function of the local oscillator amplitude if this local oscillator amplitude is about 0.6 V, corresponding to the minimum lo power needed to drive the mixer. Fig. 3 shows that the intermod amplitude is a near perfect linear function of the rf amplitude, with some compression present for rf amplitudes greater than 0.4 V. Note that the mixer conversion factor is about 0.55, which is close to the theoretical maximum of 0.64 (2 divided by Pi).

## V. Conclusion

The theory, the simulations and the experiments show that the rational models based upon the describing function approach provide better models than the Volterra theory for modelling hard nonlinearities in the frequency domain. It is illustrated how the models can be used in order to understand the mixer behavior.

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