

“Network Analysis Beyond S-parameters”

Jan Verspecht, F. Verbeyst and M. Vanden Bossche

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# Black Box Modelling of Power Transistors in the Frequency Domain

Jan Verspecht

Hewlett-Packard Network Measurement and Description Group  
VUB-ELEC, Pleinlaan 2, 1050 Brussels, Belgium  
tel. 32-2-629.2886, fax 32-2-629.2850, email janv@james.belgium.hp.com

## Abstract

**A frequency domain black box model for power transistors is proposed. It can accurately predict the behavior of the transistor for a one tone excitation with arbitrary fundamental and harmonic impedances present at the output. The model parameters can be extracted out of a limited set of “nonlinear network analyzer” measurements. Both simulated as well as measured results are given.**

## Introduction

It is not easy to design a good microwave power amplifier. Powerful CAD tools save a lot of time. It is important, however, to be aware that these simulators can only be as accurate as the mathematical models that are used. As a consequence a lot of time is spend on constructing good models for the components used. Especially constructing models that can accurately describe the large-signal hard-nonlinear behavior of power transistors is far from trivial. The state-of-the-art is to use technology dependent analytical models (e.g. Curtice Cubic, Materka, Statz, Tajima,...) or more general “small signal measurement based” models like the HP-Root model [1].

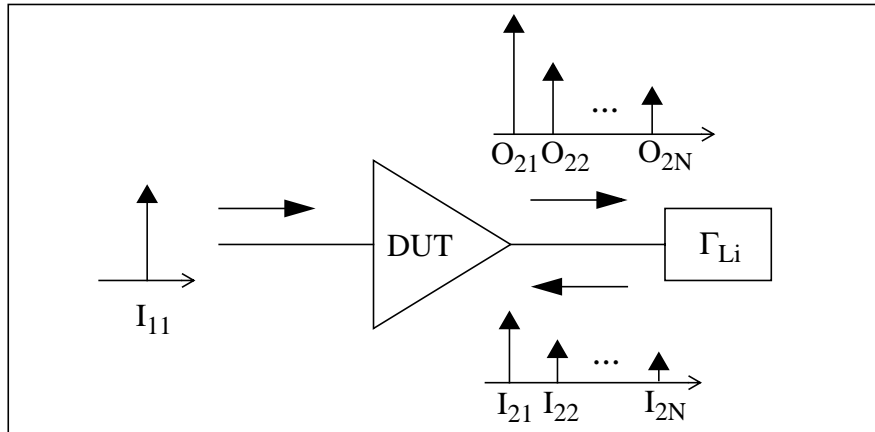
In this work another approach is proposed, based upon the use of a black-box frequency domain model. The model can simulate the behavior of a power transistor under large-signal one-tone excitation at the input, with any arbitrary impedances present at the output (fundamental and all harmonics). The model parameters are extracted based upon a relatively small set of measurements performed with a vectorial “nonlinear network” analyzer [2]. There are mainly two reasons why one can expect this approach to be more simple and accurate. Firstly one has the advantage that the model parameters are directly extracted from large signal measurements, which are the actual working conditions of the device, while the other models are based upon many small signal and DC measurements. Secondly all parasitic effects are automatically included in the black-box model, while all parasitic effects have to be explicitly identified with the other models. A drawback of the method is of course that the model will only be valid for one-tone excitation, with a frequency corresponding to the frequency used to extract the model.

## Theory

In the following the theory of the modelling approach is explained.

Suppose one has a device-under-test, called DUT, (typically one power transistor, but the theory can be applied on a whole power amplifier containing several transistors) excited by a large single-tone signal at the input and with arbitrary impedances (fundamental and harmonics) present at the output. This is depicted in Fig. 1.

Fig. 1 Schematic of the device-under-test



In this figure  $I_{11}$  denotes the complex number representing the single-tone voltage wave incident to the input,  $O_{2i}$  denotes the complex number representing the  $i^{\text{th}}$  harmonic of the voltage wave scattered by the output,  $I_{1i}$  denotes the complex number representing the  $i^{\text{th}}$  harmonic of the voltage wave incident to the output, while  $\Gamma_{Li}$  is the complex number representing the reflection factor seen by the  $i^{\text{th}}$  harmonic at the output. For simplicity, the phases of all spectral components will be defined relative to the  $I_{11}$  signal, which is used as a kind of time reference signal in order to define the phases of the other components (as a consequence  $I_{11}$  will have an imaginary part equal to zero). Note that the voltage waves are defined in a characteristic impedance which is typically 50 Ohms. The problem is to find a model for the DUT, based upon a limited number of measurements, which allows to describe the scattered components  $O_{21}, \dots, O_{2N}$  as a function of  $I_{11}$  and the harmonic reflection factors  $\Gamma_{Li}$  ( $N$  denotes the number of significant harmonics). This is done by identifying a black-box model which describes these scattered components  $O_{21}, \dots, O_{2N}$  as an analytical function of all incident components  $I_{11}$  and  $I_{21}, \dots, I_{2N}$  and their conjugates [3]. This is illustrated in Eq. 1,

$$O_{2i} = F_i(I_{11}, I_{21}, \dots, I_{2N}, I_{11}^+, I_{21}^+, \dots, I_{2N}^+) \quad \text{Eq. 1}$$

where  $F_i$  stands for the describing function corresponding to the  $i^{\text{th}}$  harmonic and where the superscript “+” stands for the conjugate.

The solution for  $O_{21}, \dots, O_{2N}$  can then be found by solving the combined set of Eq. 1 and Eq. 2 in the  $2N$  unknowns  $I_{21}, \dots, I_{2N}$  and  $O_{21}, \dots, O_{2N}$ , where

$$I_{2i} = \Gamma_{Li} O_{2i} \quad \text{Eq. 2}$$

describes the reflection of the voltage waves  $O_{21}, \dots, O_{2N}$  caused by the fundamental and harmonic mismatches present at the output of the DUT.

In order to simplify the identification of the describing functions, it is assumed that these functions behave linearly versus the components  $I_{22}, \dots, I_{2N}$ . These components are assumed to be relatively small signal components because of Eq. 2, the fact that the amplitude of the harmonic components  $O_{22}, \dots, O_{2N}$  is significantly smaller than  $O_{21}$  (the fundamental) and the amplitude of the reflection factors  $\Gamma_{Li}$  is smaller than 1 (the terminations are assumed to be passive). The final mathematical model then becomes:

$$O_{2i} = K_i(I_{11}, I_{21}) + \sum_{j=2}^N L_{ij}(I_{11}, I_{21}) I_{2j} + \sum_{j=2}^N M_{ij}(I_{11}, I_{21}) I_{2j}^+, \quad \text{Eq. 3}$$

where  $K_i$ ,  $L_{ij}$  and  $M_{ij}$  represent the linearizing complex coefficients, which are a function of the

large signal inputs  $I_{11}$  and  $I_{21}$  (for simplicity these functions are allowed to be non-analytical, such that the complex conjugate of the large signal inputs can be omitted in the mathematical representation).

The value of the coefficients, for a particular value of  $I_{11}$  and  $I_{21}$ , is found by performing an experiment where one excites the DUT several times with a linearly independent set of voltage waves  $I_{22}, \dots, I_{2N}$ . This experiment can be repeated for a grid of values  $I_{11}$  and  $I_{21}$ , such that interpolation or curve fitting can be used to find the coefficients for values of  $I_{11}$  and  $I_{21}$  which are not on the measured grid.

## Simulations

Next, the above theory will be illustrated by some large signal harmonic balance simulations [4]. For this purpose the linearized black box modelling technique is tested on simulations of a class AB power amplifier (fundamental frequency of the one tone excitation is 2.5 GHz). The amplifier has one NE900200 MESFET transistor, modelled by a so-called Curtice Cubic empirical model, in a common source configuration, with a gate bias of -2.75 V and a drain bias of 6 V.

A linearized black box model (Eq. 3) is identified for this amplifier. For simplicity, the power of the one tone excitation at the input is fixed at 15 dBm, corresponding to a fixed complex value of 1.778 Volts for  $I_{11}$  (peak values are used for frequency domain representations). A grid of 144 complex values is chosen for  $I_{21}$ , with 12 uniformly distributed values chosen as well for the real part of  $I_{21}$  (noted  $\text{Re}(I_{21})$ ) as for the imaginary part (noted  $\text{Im}(I_{21})$ ). The chosen values range from -6 V to +6 V. At each grid point for  $I_{21}$   $K_i$ ,  $L_{ij}$  and  $M_{ij}$  (Eq. 3) are identified with  $N$ , the number of harmonics considered, equal to 7. The model finally uses linear interpolation in order to find the values  $K_i$ ,  $L_{ij}$  and  $M_{ij}$  for  $I_{21}$  values which lie in between measured grid points.

Large signal harmonic balance simulations of loadpull experiments are then performed. The results of these simulations (based upon the Curtice Cubic empirical model) are then compared with the results of the same harmonic loadpull experiment, but based upon the linearized black box model.

In a first simulation a simple fundamental loadpull experiment is performed, with all harmonics terminated by a perfect match. The goal is to illustrate how well the linear interpolation works, based upon the 144 complex values of  $I_{21}$  used for the measurements. The result is shown in Fig. 2. This figure shows the complex value of  $O_{21}$  as a function of  $\Gamma_{L1}$ . For the simulation three amplitudes of  $\Gamma_{L1}$  are chosen, namely 0.2, 0.6 and 1.0, while 25 values are chosen for the argument, ranging from  $-\pi$  to  $+\pi$  rad.

A second simulation is used to demonstrate the capability of the black box model to simulate the fundamental as well as the harmonic loadpull behavior of the component. For this purpose  $\Gamma_{L1}$  is made equal to a fixed  $+j$  (notation used for  $\sqrt{-1}$ ), and the dependency of  $O_{21}$  versus  $\Gamma_{L2}$  is simulated. The same values for  $\Gamma_{L2}$  are chosen as were used for  $\Gamma_{L1}$  in the first simulation. The results are shown in Fig. 3.

As can be seen in Fig. 2 and Fig. 3 there is a very good correspondence between fundamental and harmonic loadpull behavior of the black box linearized model and the harmonic balance simulations, based upon the Curtice Cubic FET model. This implies that the assumption used for the linearization (cf. Eq. 3) holds very well for this case, even for extreme values of  $\Gamma_{L_i}$  (amplitudes equal to 1).

Fig. 2

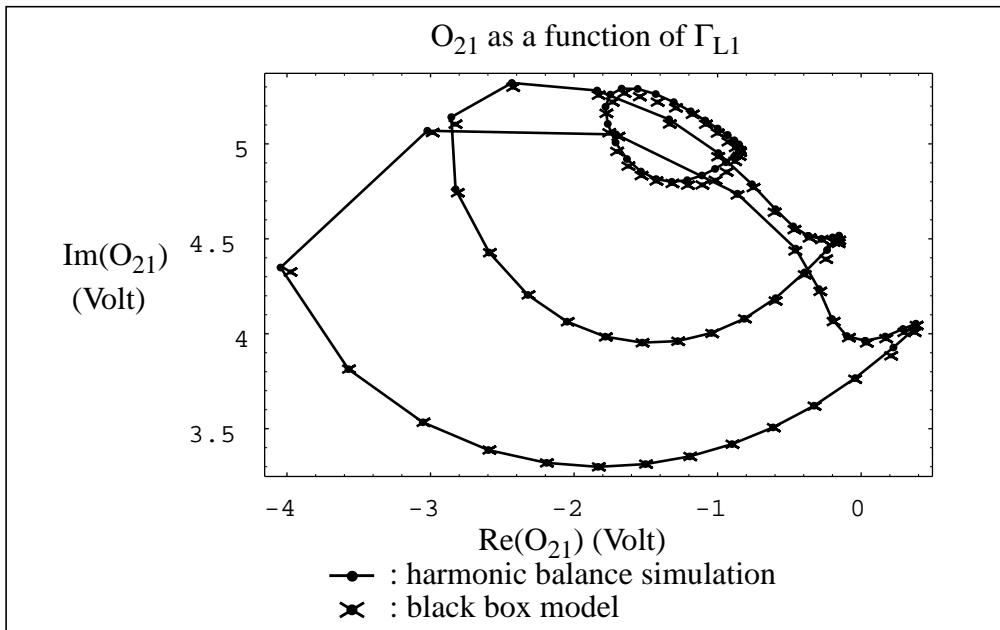
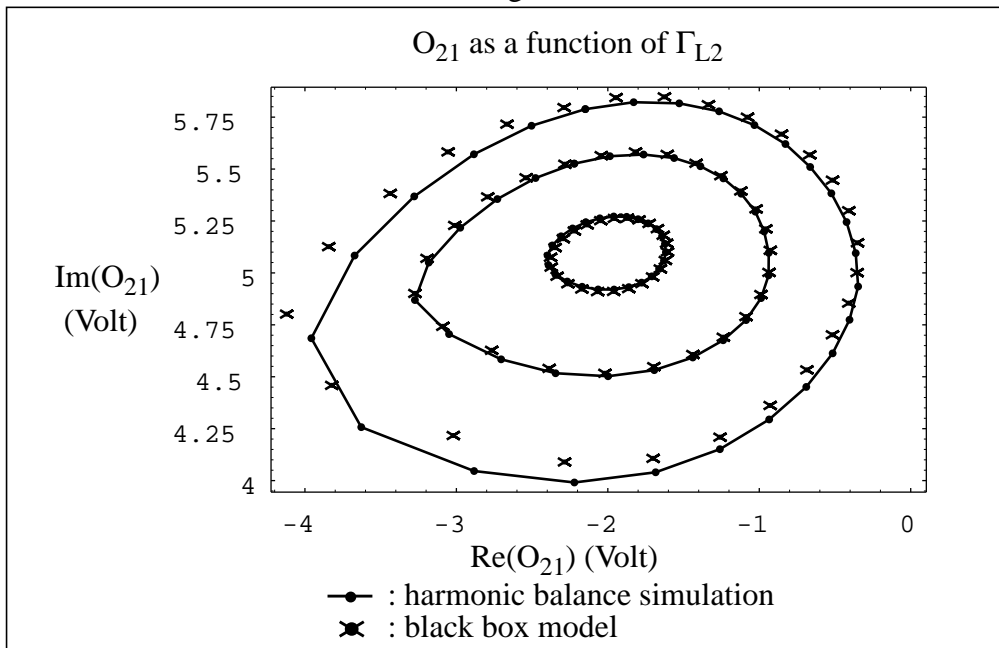


Fig. 3



## Measurements

Next the black-box linearized model approach is tested on measured data. The set-up depicted in Fig. 4 is built for this purpose. The DUT chosen is a power HFET transistor, biased for power amplification in class AB-mode (-4.4 V gate voltage, 7 V drain voltage, with  $I_{11}$  fixed and equal to 3.08 V, corresponding to an incident power of 19.8 dBm). The fundamental frequency chosen is 1.8 GHz. A tuner allows to control  $\Gamma_{L1}$ , for the measurements performed the amplitude of  $\Gamma_{L1}$  equals 0.278, with an angle of  $43.8^\circ$ . The reflection coefficients of the harmonics are all very

small. A synthesizer allows to inject a component  $I_{2j}$  towards the drain, while a “nonlinear network analyzer” [2] allows to measure the amplitude and the phases of the components  $I_{2j}$  and  $O_{2j}$ , for  $j$  going from 1 to 10 (18 GHz bandwidth).

A set of measurements is then performed where an harmonic component  $I_{22}$  is injected towards the drain, with two different amplitudes and 40 different randomly chosen phases. The set of measured  $I_{22}$ 's is depicted in Fig. 5. With the set of measured  $I_{22}$ 's corresponds a set of measured  $O_{21}$ 's, which is depicted by the squares in Fig. 6. A least-squares-error approach is then used in order to fit a linearized black-box model (Eq. 3) on the measured data. This model can then predict the  $O_{21}$ 's corresponding to the measured  $I_{22}$ 's. The modeled  $O_{21}$ 's are depicted by the crosses in Fig. 5. The correspondence between measured and modelled output is very good, which indicates that the black-box model accurately describes the transistor behavior with harmonic components being injected into the drain. The identified model for  $O_{21}$ , which is valid for the value of  $\Gamma_{L1}$  used during the experiment and for  $I_{2j} = 0$  for  $j > 2$ , is:

$$O_{21} = -4.388 + j2.308 - (0.560 + j0.029)I_{22} - (0.064 - j0.027)I_{22}^+ . \quad \text{Eq. 4}$$

Fig. 4

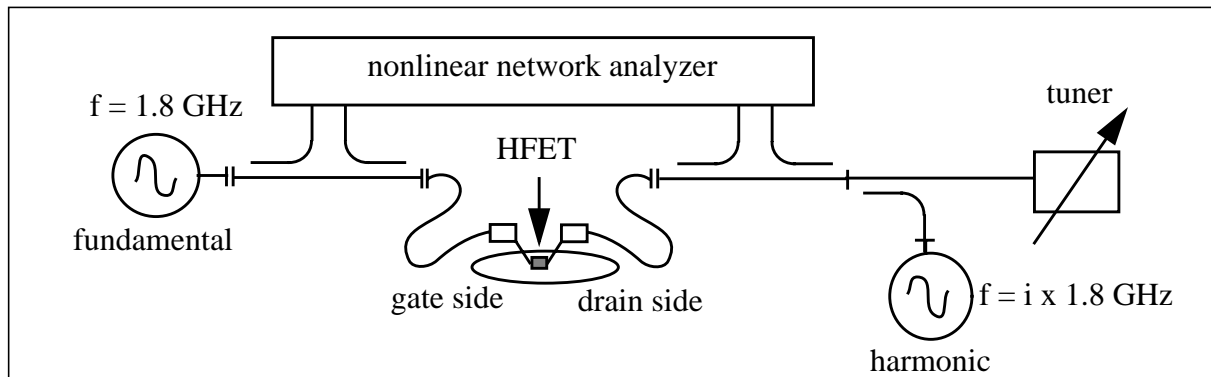


Fig. 5

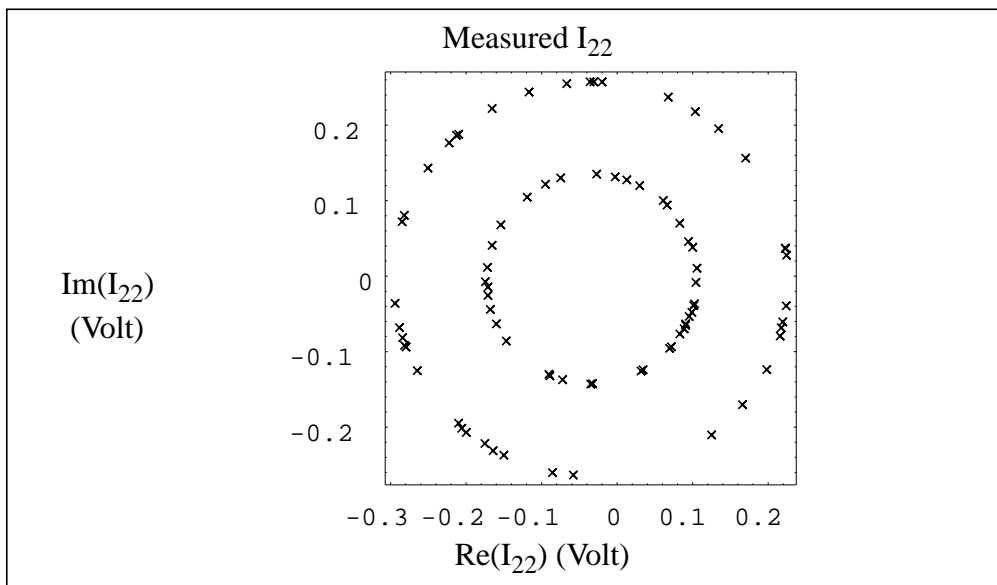
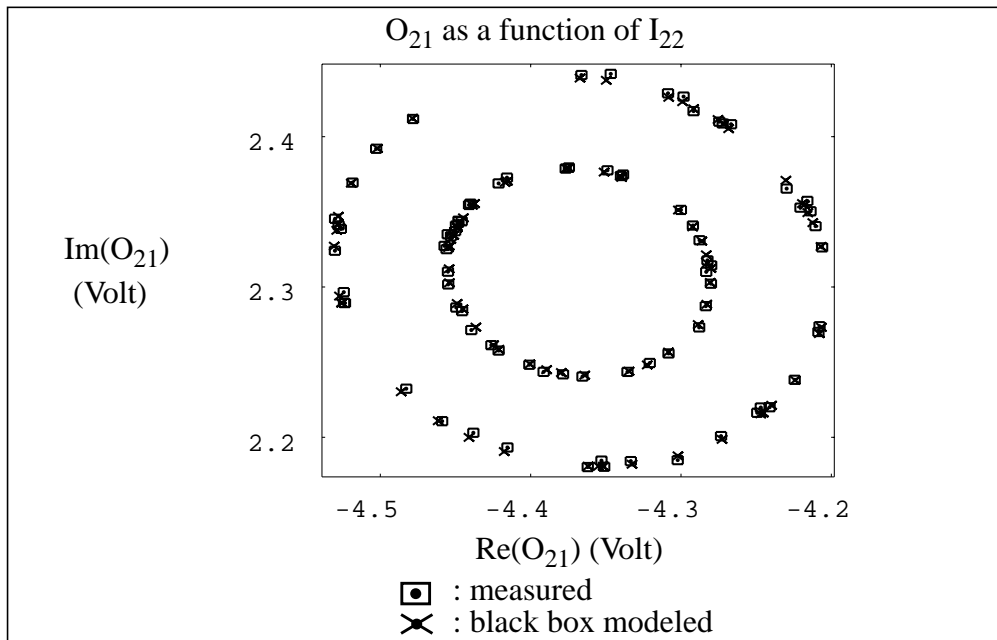


Fig. 6



## Conclusions

The simulations and measurements show that the theoretically developed linearized black-box model accurately models the behavior of a power transistor when harmonic voltage waves are injected towards the output terminal (these voltage waves can be caused by harmonic mismatches or by external sources). Using vectorial nonlinear network analyzer measurements the black box model parameters describing the behavior of an actual device can accurately be determined.

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