Signal Characterization and Modulation Theory

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Spectral efficiency is of paramount importance when considering the design of virtually all commercial wireless communication systems, whether for voice, video, or data. Spectral efficiency can be measured by the number of users per unit of spect rum or by the number of bits that can be represented per unit of spectrum. In general, wireless service providers are interested in maximizing both the number of users and the number of bits per unit spectrum, which in both cases results in the maximum revenue.

Spectral efficiency can be increased by using several methods including signal polarization, access method, modulation method, and signal coding technique. The first method is commonly adopted in satellite communication systems, where, for example, the uplink and downlink may be right-hand and left-hand circularly polarized, respectively.

Access method refers to how a common resource, such as frequency, is shared among each of the users of the system. Frequency domain multiple access (FDMA) is the basis for all commercial broadcast and most wireless communication systems. With FDMA, each user is allocated a particular section of spectrum that is devoted to that user only. The first-generation cellular phone system in the United States, called advanced mobile phone system (AMPS) has approximately 600 30-kHz channels, each one of which can be utilized for voice and low data-rate communication[1].

Users can also be allocated a certain segment of time, called a slot, leading to time-domain multiple access (TDMA). With TDMA, users share a common frequency and are assigned one, or in some cases more than one slot out of several available slots. In this fashion spectral efficiency is increased by segregating users in time. Second-generation wireless communication systems based on TDMA include the North American Digital Cellular system (NADC) and the Global Standard for Mobile Communications (GSM), which have three slots and eight slots, respectively[2, 3].

Code-division multiple access (CDMA) results when each user is assigned a unique code, which is orthogonal to all the other available codes. The Walsh function has been adopted as the orthogonal code set for first-generation CDMA systems. At the receiver, the signal is correlated with a known Walsh function, and the transmitted information thus extracted. The ability of these systems to improve spectral efficiency relies on the development of robust orthogonal functions, since any cross-correlation results in performance degradation. Many wireless systems are hybrids of two access methods. For example, the first CDMA-based wireless system, developed by Qualcomm, is based on FDMA and CDMA[4].

Modulation is the process of impressing an information source on a carrier signal. Three characteristics of a signal can be modulated: amplitude, frequency, and phase. In many types of modulation, two characteristics are modulated simultaneously. Analog modulation results when the relationship between the information source and the modulated signal is continuous. Digital modulation results when the modulated characteristic assumes certain prescribed discrete states.

Signal coding techniques are many and varied, and require detail beyond what can be presented in this introductory chapter. Coding can impact the characteristics of the signal, as will be examined in more detail with CDMA.

The purpose of this chapter is to provide an introduction to the representation and characterization of signals used in contemporary wireless communication systems. The time domain representation of an information-bearing signal determines uniquely what the impact of nonlinear amplification will be, so the study begins with a review of time domain signal analysis techniques. Since the effects of nonlinear amplification are of most interest in the frequency domain, the signal analysis review will also include frequency domain methods. Random process theory is an integral element of digital modulation theory, and will also be covered. Following this, a review of several types of modulations will be given, with both a time domain and frequency domain complex envelope description. The impact of filtering, for spectral efficiency improvement, will be assessed. A probabilistic time domain method of characterizing the envelope of a signal, called the envelope distribution function (EDF), will be introduced. This function is more useful than the peak-to-average ratio in estimating the impact of a PA on a signal. A complete reference section is given at the end of the chapter.

Complex Envelope Representation of Signals

Using Fourier analysis, any periodic signal can be exactly represented as an infinite summation of harmonic phasors[5]. Most often a signal *x(t)* is approximated as a finite summation of harmonic phasors, in which case

$$x(t) = \sum_{k=-\infty}^{\infty} \bar{a}_k e^{jk\omega t} = \sum_{k=-Q}^{Q} \bar{a}_k e^{jk\omega t} \qquad \textbf{(3.89a)}$$

where

$$\bar{a}_k = \frac{1}{T}\int_{t}^{t+T} x(\tau) e^{-j2\pi f \tau} d\tau \qquad \textbf{(3.89b)}$$

and *Q* is chosen sufficiently large to accurately represent the signal under consideration. A physical basis for this approximation is that all systems have an essentially low-pass response.

Parsavel's theorem states that average power is invariant with respect to which domain it is calculated in, and is expressed as

$$m(t)\cos(2\pi f_o t) \Rightarrow \frac{1}{2}\left[M(f-f_o)+M(f+f_o)\right] \qquad \textbf{(3.90)}$$

In many instances, calculation of average power may be easier to evaluate in either the time domain or the frequency domain representation.

The modulation property illustrates the frequency-shifting nature of time domain multiplication. This is expressed as

$$x(t) = real\left\{m(t)\exp\left[2\pi f_o t + \phi(t)\right]\right\} \qquad \textbf{(3.91)}$$

where *M(f)* is the Fourier transform of *m(t)*. Note that both upper and lower sidebands are generated, indicating the presence of a negative frequency component.

In many instances, knowledge of the envelope of a signal is sufficient for its characterization and for assessing the associated impact of a nonlinear PA. An arbitrarily modulated signal can be represented in the time-domain as

$$x(t) = real\left\{m(t)\exp\left[2\pi f_o t + \phi(t)\right]\right\} \qquad \textbf{(3.92)}$$

where *m(t)* and *φ(t)* describe the time-varying amplitude and phase of the information signal, respectively, and f_o is the carrier frequency[6]. Note that frequency modulation results by differentiation of the phase modulation. The complex envelope of **Eq. (3.92)** is

$$\tilde{x}(t) = i(t) + jq(t) = m(t)e^{j\phi(t)} \qquad \textbf{(3.93)}$$

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where $i(t)$ and $q(t)$ are defined as the in-phase and quadrature components of the complex envelope. Although there are other methods available of representing modulated signals, **Eq. (3.93)** is adopted here due to the elegant geometric interpretation afforded by the complex plane representation. The components of **Eq. (3.93)** are calculated from

$$i(t) = real\left\{x(t)e^{-j2\pi f_c t}\right\} \quad \textbf{(3.94a)}$$

$$q(t) = imag\left\{\hat{x}(t)e^{-j2\pi f_c t}\right\} \quad \textbf{(3.94b)}$$

where $x(t)$ is the Hilbert transform of $x(t)$[7]. Since statistically independent signals are orthogonal, calculation of the Hilbert transform is often unnecessary, and instead two statistically independent data sources for i, (t) and $q(t)$ can be used. **Figure 1** shows the spectrum of an arbitrary complex envelope equivalent power density spectrum. Note that the spectrum is not even-symmetric about the y-axis, with the resultant requirement that the time domain signal **Eq. (3.93)** is in general complex.

Linear network analysis using the complex envelope is similar to conventional network analysis. Taking the Fourier transform of **Eq. (3.93)**, and denoting the complex transfer function as $H(f)$, we have:

$$\tilde{Y}(f) = \tilde{H}(f)\tilde{X}(f) \quad \textbf{(3.95a)}$$

This response can also be calculated directly in the time domain using the convolution integral

$$\tilde{y}(t) = \int_{-\infty}^{\infty} \tilde{h}(\tau)\tilde{x}(t-\tau)d\tau \quad \textbf{(3.95b)}$$

The equivalent band-pass time domain response is found by

$$y(t) = real\left\{\tilde{y}(t)e^{j2\pi f_c t}\right\} \quad \textbf{(3.96)}$$

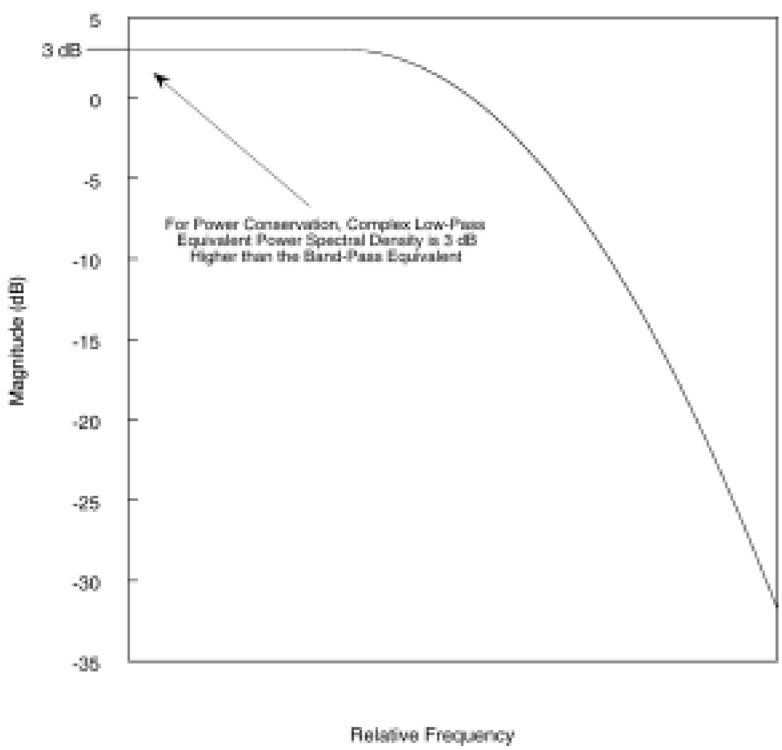

Figure 1. Power Spectral Density of an Arbitrary Complex Envelope Equivalent Signal.

Representation and Characterization of Random Signals

Consider a random signal *x(t)* that could describe either a voltage or current. In general the associated *n-dimensional* joint probability density function is required to describe *x(t)* over *n* time instants[8]. Since the expected amplitude and power of a signal are most often of interest, the first- and second- moments are sufficient for representation and characterization of *x(t)*. The first- and second-moments of *x(t)* are

$$\overline{x}(t) = \int_{-\infty}^{\infty} \tau f(\tau) d\tau = \sqrt{DC\ Power} \quad \textbf{(3.97a)}$$

(3.97b)

where $f(\tau)$ is the associated probability distribution function (pdf) of *x(t)*. The pdf describes the probability of the instantaneous amplitude of *x(t)* being less than a specified value. This idea is illustrated with a Gaussian pdf in **Figure 2**. Note that the amplitude of this signal is concentrated around its mean value, which is zero.

Since the average value and average power of *x(t)*, as determined by **Eq. (3.97)**, are based on ensembles, they are defined as ensemble averages. The average value and average power of a signal can also be calculated with respect to time, which is the method most engineers are familiar with. Signals are said to be ergodic when their ensemble and time averages are the same. All of the signals described in this chapter are ergodic.

Probabilistic characterization of how rapidly a signal changes over time, and hence its spectral distribution, is described by its autocorrelation function

$$R_x(\tau) = \overline{x(t)x(t+\tau)} = \iint_{-\infty} \tau_1 \tau_2 f(\tau_1, \tau_2) d\tau_1 d\tau_2 \quad \textbf{(3.98)}$$

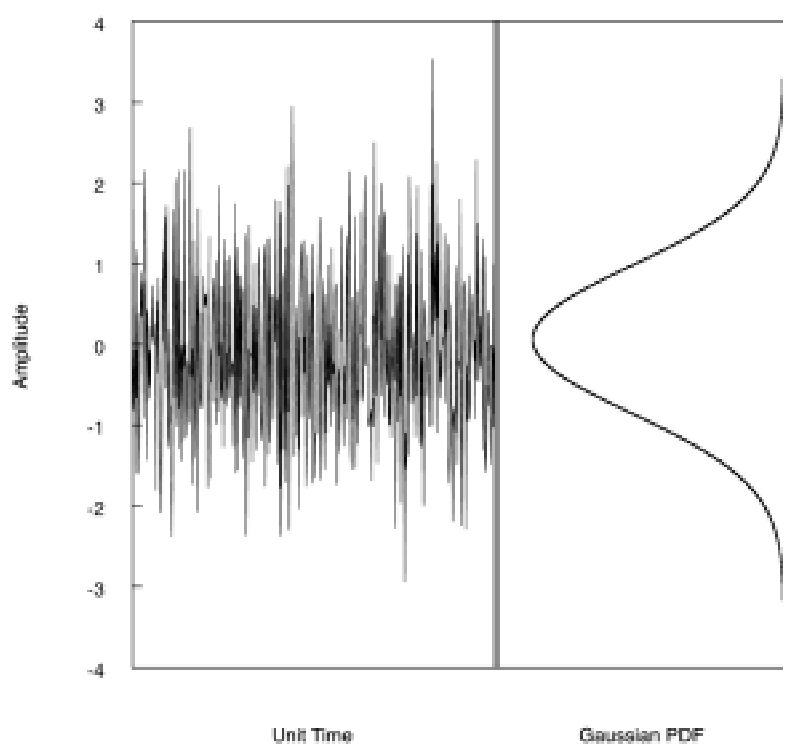

Figure 2. Illustration of the Meaning of Random Signal Having a Gaussian Amplitude Distribution. In this Example the Gaussian pdf Represents the Probability of the Signal Amplitude being Less than a Given Value. For Example, since the pdf is Symmetric about the Time Axis, this Signal has Zero Mean. Since the Coise in this Example is Uncorrelated, it is Defined as White. Similarly, a Colored Noise Process will Exhibit Correlation.

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The autocorrelation function evaluated at $\tau = 0$ is the total average power in the signal, as **Eq. (3.97b)** shows. Using **Eq. (3.97a)** and **Eq. (3.97b)**, which represent average amplitude and total average power, respectively, gives the AC power of the signal

$$P_{AC} = R_x(0) - (\bar{x})^2 = \overline{x^2} - (\bar{x})^2 \quad \textbf{(3 99)}$$

Using the Wiener-Khintchine theorem, the spectral distribution of power of *x(t)* is evaluated by taking the Fourier transform of the autocorrelation function

$$S_x(f) = \int_{-\infty}^{\infty} R_x(\tau) e^{-j2\pi f\tau} d\tau \quad \textbf{(3.100)}$$

where $S_x(f)$ is the power spectral density (PSD) of *x(t)*.

In the case of an arbitrary autocorrelation function, $Rx(\tau)$, the PSD is expressed as

$$S_x(f) = S_{pulse}(f) S_{corr}(f) \quad \textbf{(3.101)}$$

where $S_{pulse}(f)$ is the PSD of the pulse function representing *x(t)* and $S_{Corr}(f)$ is the PSD of the autocorrelation function of the data[9]. From this expression, it is clear that the spectral characteristics of a signal are influenced not only by the pulse characteristics, but also any correlation between adjacent pulses.

Let *x(t)* be the input to a linear system and let *y(t)* be the associated output. The impulse response, *h(t)*, is described in the frequency domain as *H(f)*. The average amplitude of *y(t)* is

$$\bar{y}(t) = \bar{H}(0)\bar{x}(t) \quad \textbf{(3.102)}$$

and the average power of *y(t)* is

$$\overline{y^2}(t) = R_y(0) = \int_{-\infty}^{\infty} |H(f)|^2 df = \int_{-\infty}^{\infty} |H(f)|^2 S_x(f) df \quad \textbf{(3.103)}$$

The output autocorrelation function is

$$R_y(\tau) = h(\tau) * h(-\tau) * R_x(\tau) \quad \textbf{(3.104)}$$

where * denotes convolution. Taking the Fourier transform gives the output PSD in terms of the input PSD

$$S_y(f) = |H(f)|^2 S_x(f) \quad \textbf{(3.105)}$$

Complex envelope equivalent analysis is done by replacing all variables with the associated complex envelope representation and restricting the integration of **Eq. (3.103)** to positive frequency[6]. Note that this analysis gives an example of how choosing the domain in which to carry out an analysis can greatly simplify the effort involved.

Modulation Theory

Modulation theory provides a framework for representing and characterizing the time domain and frequency domain characteristics of an information-bearing signal, and the subsequent impact of non-linear amplification of the signal. The general analysis method to be followed is to describe the signal in the time domain, using the geometric interpretation of **Eq. (3.93)**, and then determine resultant signal degradation by characterization in the frequency domain.

Access method, modulation, and coding each directly impact the spectral efficiency of a signal. From Fourier analysis it is also clear, therefore, that the time domain characteristics are also impacted, due to the inverse relationship between the time domain and frequency domain. In other words, a signal that varies rapidly in time, due to access method, modulation, or coding, will be wider in extent in the frequency domain than a slowly varying signal. Note here that signal refers to the envelope of the carrier, and the extent of the signal in the frequency domain refers to the associated spectral description of the envelope.

As **Eq. (3.93)** indicates, modulation can be interpreted geometrically by associating a position in the complex plane with the instantaneous value of the information source. For analog modulation, a continuous range of values is possible digital modulation allows only certain locations to be occupied. This mapping operation will directly establish the resultant time domain characteristics of the modulation. Many modulations are based on phase since they are relatively impervious to amplitude-related noise disturbances, FM being an example of this. When phase modulation is digital, the characteristics of the digital pulse will influence the signal as well, as **Eq. (3.101)** shows. Since pulses may be rapidly varying, it is expected that some type of filtering will be necessary to give acceptable spectral efficiency. Thus, to describe a modulation, the mapping method and any associated band-limiting filtering must be considered to provide a complete time domain description of the complex envelope.

Analog Modulation

The simplest modulation is analog double sideband suppressed carrier (DSB-SC). When the information source is a sinusoid, DSB-SC is the classical two-tone intermodulation test signal. The complex envelope representation of DSB-SC is

$$\tilde{x}(t) = m(t) \qquad \textbf{(3.106)}$$

where *m(t)* is the information signal. Since the envelope of DSB-SC varies in direct proportion to *m(t)*, it is not a constant envelope modulation. Note also that, from a geometric interpretation using **Eq. (3.93)**, DSB-SC requires only one dimension, meaning there is no phase modulation.

All of the first-generation cellular systems, such as AMPS and ETACS, are based on FM, with a complex envelope representation of

$$\tilde{x}(t) = \exp\left[jk_f \int_{-\infty}^{t} m(\tau)\, d\tau \right] \qquad \textbf{(3.107)}$$

where k_f is the frequency-deviation constant and *m(t)* is the information signal. Since the magnitude of the complex exponential is unity, FM is a constant envelope modulation. A geometric interpretation of FM shows that it would be a unit circle, with the speed in movement about this circle proportional to the instantaneous value of the information signal. Carson's rule shows that the spectral efficiency of FM is less than DSB-SC 6. In general, constant envelope modulation is not as spectrally efficient as modulation that exhibits a time-varying envelope.

Discontinuous Phase-Shift Keying

Virtually all second-and third-generation wireless communication systems are based on digital modulation, with digital phase modulation being the most common. Digital phase modulation is commonly referred to as phase-shift keying (PSK). PSK consists of two DSB-SC signals in quadrature, and can be represented using **Eq. (3.93)** with in-phase and quadrature components each generated from a digital data source. Each data source is usually generated by multiplexing a serial data stream, which, if necessary, is already coded, such as with CDMA Quadrature digital modulation is represented in the complex plane as shown in **Figure 3**, where the horizontal axis represents the real part of **Eq. (3.93)** and the vertical axis represents the imaginary part of **Eq. (3.93)**. The trajectory is the instantaneous envelope of the signal and for the constellation given, there are four unique phases, with each phase representing a unique combination of two bits. Each combination of bits is defined as a symbol. Some third-generation wireless systems have adopted PSK with eight unique locations, giving 8-PSK, with the result that each location now represents a unique combination of three bits. Note also from **Figure 3** that PSK modulation is constant envelope.

The complex envelope representation of quadrature digital modulation is

$$\tilde{x}(kT_b) = i(kT_b) + jq(kT_b) \qquad \textbf{(3.108)}$$

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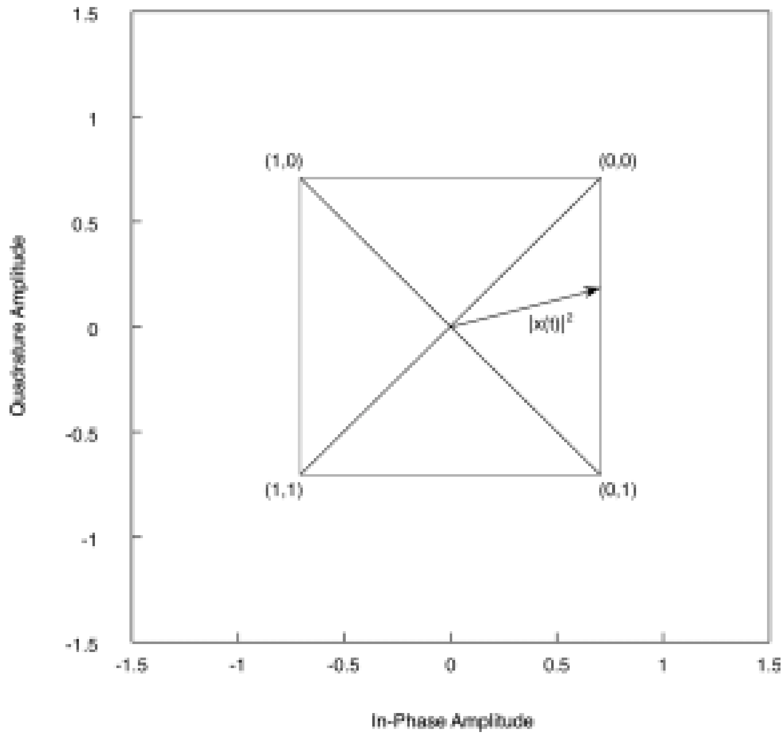

Figure 3. Complex Plane Representation of Unfiltered Quadrature Modulation (QPSK). Each Corner of the Constellation Diagram Represents Two Data Bits. Depending on the Data Sequence, each Transition goes Through a Phase Change of Either ± 90° or ± 180°, which Always Causes a Step Change in the Signal Envelope.

where

$$i(kT_b) = \sum_{k=-\infty}^{\infty} a(kT_b) f(t - kT_b - \tau_f) \quad \textbf{(3.109a)}$$

$$q(kT_b) = \sum_{k=-\infty}^{\infty} b(kT_b) g(t - kT_b - \tau_g) \quad \textbf{(3.109b)}$$

and *a(kT$_b$)* and *a(kT$_b$)* represent unit-amplitude data sources, *f* and *g* are pulse functions, T_b is the bit rate, and τf and τg are arbitrary phase offsets. The functions *f* and *g* are usually rectangular pulse streams with zero mean and zero correlation between pulses. In this case, the PSD of **Eqs. (3.109a)** and **(3.109b)** is given as

$$S_i(f) = T_b \text{sinc}^2(\pi T_b f) \quad \textbf{(3.10 9c)}$$

$$S_q(f) = T_b \text{sinc}^2(\pi T_b f) \quad \textbf{(3.109d)}$$

In **Figure 4** it is shown that **Eqs. (3.109c)** and **(3.109d)** exhibit a-6 dB/octave roll off. A communication system based on rectangular pulses would therefore exhibit very poor spectral efficiency due to the relatively large energy present in the spectrum in the neighboring sidelobes. To resolve this problem, band-limiting filtering is used. Although it is possible to use arbitrary band-limiting filters, such as the Chebyshev low-pass response, the Nyquist response is often adopted since it exhibits certain characteristics amenable to demodulation of the signal at the receiver[10-12].

A significant consequence of band-limiting a discontinuous signal, such as that exhibited by PSK with rectangular pulses, is that envelope variations will be introduced. Thus, although PSK modulation is constant envelope, the necessity of band-limiting induces envelope variations, which gives a band-limited PSK signal properties of both phase modulation and amplitude modulation. The degree of amplitude variation is directly proportional to the degree of band limiting. **Figure 5** illustrates the impact of band limiting on the QPSK constellation of **Figure 3**, where the amplitude variations are apparent. Band limiting will necessarily require a reduction in the efficiency of a PA, in order to support the peak excursions of the envelope.

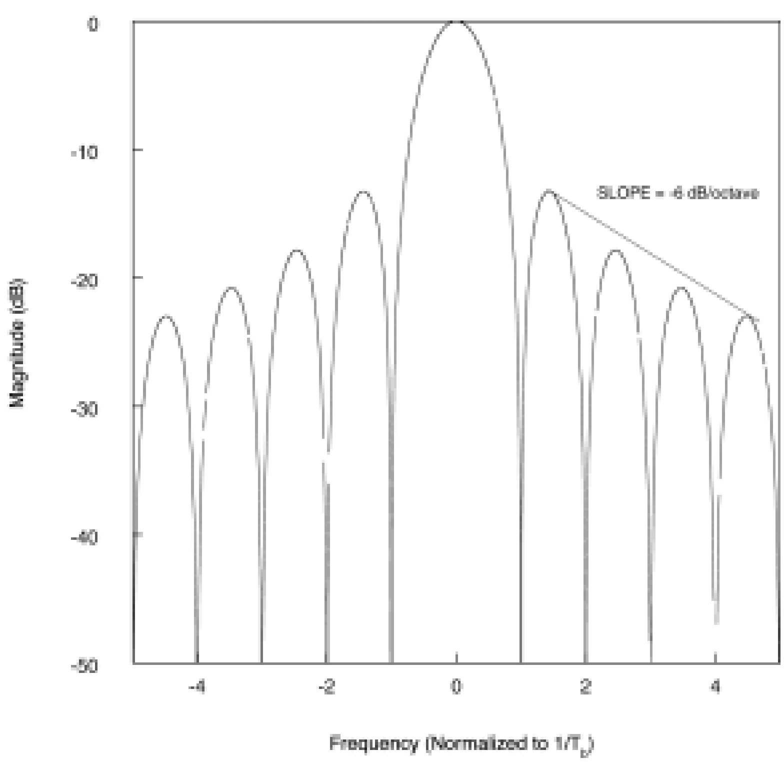

Figure 4.

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The Nyquist filter response exhibits simultaneous band limiting and zero inter-symbol interference (ISI). Zero ISI is desirable, though not necessary, for a digital communication system to give acceptable performance. To understand the impact of inter-symbol interference, consider the convolution integral **Eq. (3.95b)**, where it is seen that convolution is essentially a summation operation that accounts for the past state of a signal or system at the present time. Use of an arbitrary filter response will result in interference of the present state of a signal due to its past values, leading to the potential for an error in the actual value of the signal at the present time. The impulse response of a Nyquist filter has zero crossings at multiples of the symbol rate, and thus does not cause inter-symbol interference[11, 12].

The most common form of Nyquist filter currently adopted is the raised cosine response. The raised cosine response is expressed in the frequency domain as

$$H(f)=\begin{cases} T_s, & |f| \le \frac{1}{2T_s}-\alpha \\ T_s\cos^2\left[\frac{\pi}{4\alpha}\left(|f|-\frac{1}{2T_s}+\alpha\right)\right], & \frac{1}{2T_s}-\alpha \le |f| \le \frac{1}{2T_s}+\alpha \\ 0, & |f| > \frac{1}{2T_s}+\alpha \end{cases} \quad (3.110)$$

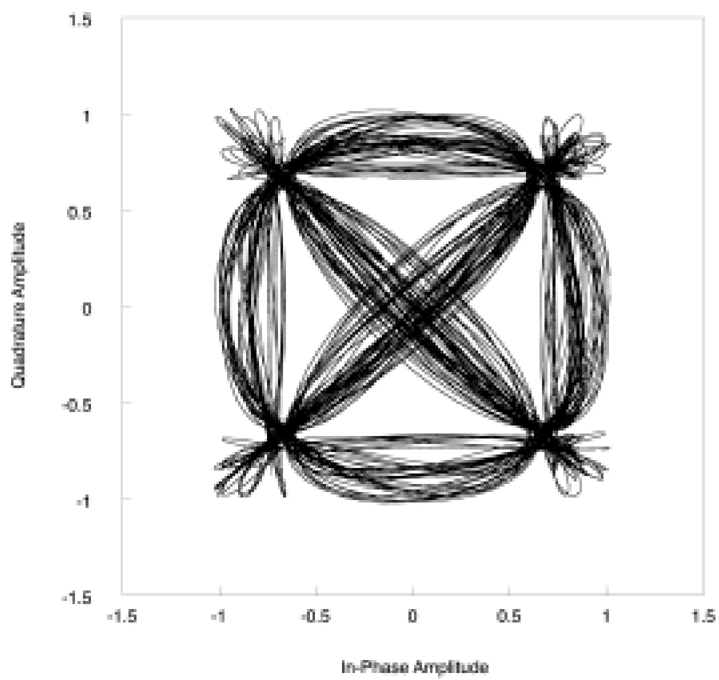

Figure 5. Constellation Diagram of Band-limited QPSK. The Transient Response of the Band-limiting Filter Increases the Peak-Average Ratio of Signal. The more Severe the Band Limiting, Yielding Increased Spectral Efficiency, the Higher the Peak-to-average Ratio Becomes.

where T_s is the symbol rate in seconds and α is the excess-bandwidth factor[6, 13]. By adjusting α, the spectral efficiency of a digitally modulated signal can be controlled, while maintaining zero ISI. **Figure 6** shows the frequency response of **Eq. (3.110)** for various values of α. At the Nyquist limit of α=0, all energy above half of the main lobe is removed. This results in maximum spectral efficiency but the largest time domain peak-to-average ratio.

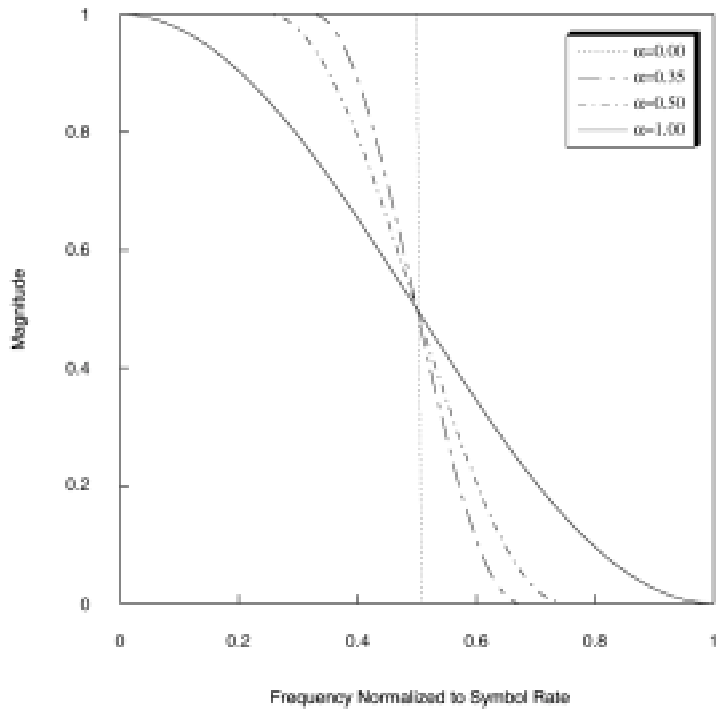

Figure 6. Raised-cosine Filter Response for Several Excess-bandwidth Factors. The Frequency Axis is Normalized to the Symbol Rate. NADC uses an α = 0.35.

To maximize receiver signal-to-noise ratio (SNR), many wireless communication systems split the filter response equally between the transmitter and receiver[9, 10]. The resultant response is called the square-root raised-cosine response, which in the time domain is expressed as

$$h(t) = 4\alpha \frac{\cos\left[(1+\alpha)\frac{\pi}{T_s}t\right] + \dfrac{\sin\left[(1-\alpha)\frac{\pi}{T_s}t\right]}{\frac{4\alpha}{T_s}t}}{\left(\pi T_s^{0.5}\right)\left[\left(\frac{4\alpha}{T_s}t\right)^2 - 1\right]} \quad \textbf{(3.111)}$$

where the variables are defined as above in **Eq. (3.110)**[11].

Many digital wireless standards specify the symbol rate and excess-bandwidth factor to specify the root-raised cosine filter. This can lead to ambiguity in specifying the filter response insofar as the length of the impulse response is not specified. An impulse response that is too short will exhibit unacceptable band-limiting performance and can lead to incorrect characterization of spectral regrowth. Alternatively, some standards specify a band-limiting impulse response in the form of a digital filter to ensure that the appropriate frequency domain response is used[4, 14].

Several wireless communication systems are based on discontinuous PSK modulation. These include the North American Digital Cellular standard (NADC), the Personal

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Digital Cellular System (PDC), the Personal Handyphone System (PHS), and cdmaOne[2, 15, 4]. The first three of these systems have adopted π/4-DQPSK modulation with a root-raised cosine filter response. The cdmaOne system has adopted QPSK for the forward-link and offset QPSK (O-QPSK) for the reverse link, in both cases using a Chebyshev filter response.

π/4-DQPSK, a derivative of QPSK, differentially encodes the symbols, and adds a π/4 rotation at each symbol instant. This results in an envelope that avoids the origin, thus decreasing the peak-average of the signal. This is done presumably to reduce the PA back-off requirement, but as recent work has illustrated, this is not achieved in practice due to the original assumptions made[16]. **Figure 7** shows the constellation diagram for π/4-DQPSK with α = 0.35, corresponding to the NADC standard. Note that at each of the constellation points, the trajectories do not overlap; this is inter- symbol interference. At the receiver, this signal will be convolved with the remaining half of the raised-cosine filter, leading to the desired response given by **Eq. (3.110)**. PDC and PHS are also based on π/4-DQPSK, with an excess bandwidth factor of α = 0.50. Their constellation diagrams are similar to the NADC diagram shown in **Figure 7**.

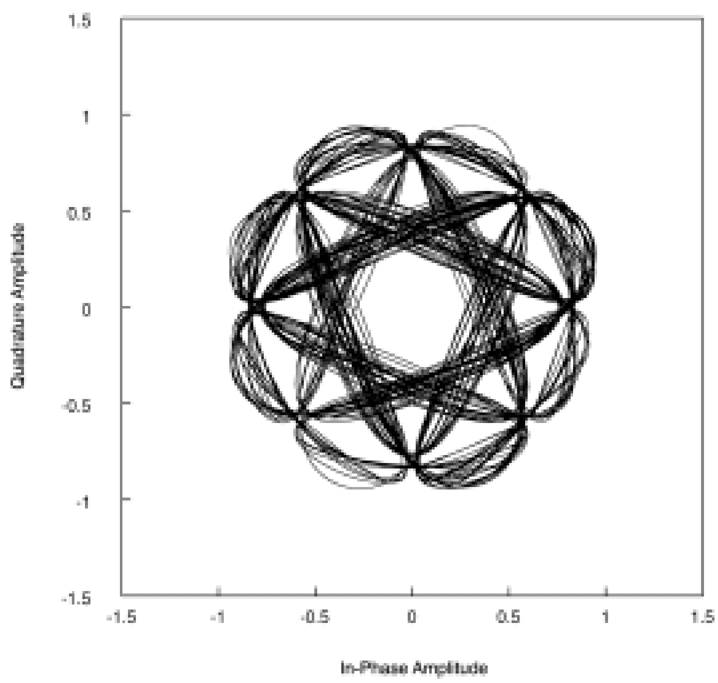

Figure 7. Constellation Diagram for π/4-DQPSK with a = 0.35. This modulation is used for NADC, PDC, and PHS.

Figure 8 shows the constellation diagram for reverse-link cdmaOne, which is based on O-QPSK. Like π/4-DQPSK, O-QPSK was developed to reduce the peak-to-average ratio of the signal. It has been shown that this approach, while reducing the peak-to-average ratio, does not necessarily reduce the PA linearity requirements 16. Note also in **Figure 8** the significant ISI, due to the fact that a Nyquist filter was not used. The forward link for cdmaOne uses QPSK modulation, which is shown in **Figure 5**. As with the reverse link, the forward link uses a Chebyshev response specified by the standard.

To increase spectral efficiency, it is possible to add four additional phase locations to QPSK, called 8PSK, with each location corresponding to a unique combination of three bits. **Figure 9** shows the constellation diagram for the EDGE standard, which is based on 8-PSK and a specially developed filter that exhibits limited response in both the time domain and frequency domain[17]. EDGE was developed to increase the system capacity of the current GSM system while minimizing the requirements for linearity. In contrast to GSM, which is constant envelope, the EDGE systems provides increased spectral efficiency at the expense of envelope variations. However, EDGE intentionally minimizes the resultant amplitude modulation to ensure that existing GSM amplification systems, optimized for constant envelope modulation, could still be used.

Continuous Phase-Shift Keying

With the exception of EDGE, each of the PSK modulations considered in the previous section exhibited significant out-of-band energy due to the discontinuous transitions. To reduce the out-of-band energy, band limiting was required, which resulted in envelope variations. An alternative modulation process

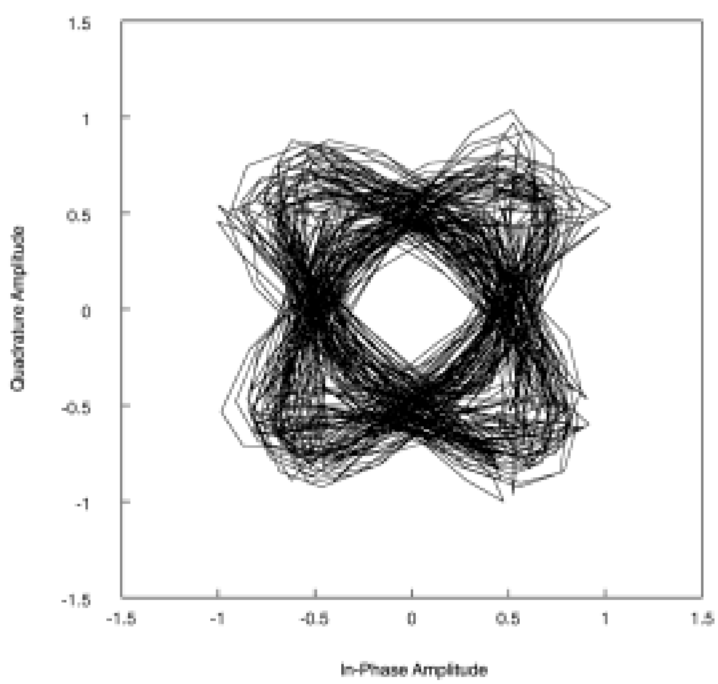

Figure 8. Constellation Diagram for O-QPSK Using the Filter Response Specified by the IS-95 Standard for cdmaOne.

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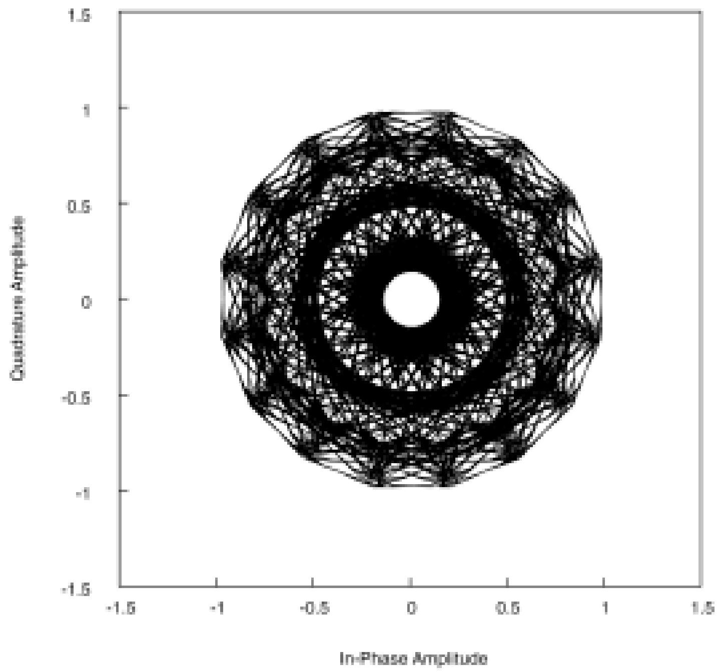

Figure 9. Constellation Diagram for EDGE.

is continuous-phase modulation (CPM), which requires that the phase transitions from one symbol to the next be continuous[19, 13]. This results in a signal that is intrinsically band limited while maintaining a constant envelope. The price paid is a potential reduction in spectral efficiency. Two types of CPM will be examined in this section: minimum-shift keying (MSK) and Gaussian MSK (GMSK).

The complex envelope representation of MSK is

$$x(t)=\cos\left[a\left(kT_b\frac{\pi t}{2T_b}\right)\right]+j\sin\left[b(kT_b)\frac{\pi t}{2T_b}\right] \qquad \textbf{(3.112)}$$

$a(kT_b)$ and $b(kT_b)$ represent unit-amplitude data sources as defined in **Eq. (3.109)**. This modulation is identical to O-QSPK using a half-wave sinusoidal pulse function instead of a rectangular pulse function. Taking the magnitude squared of each term of **Eq. (3.112)** shows that it is indeed a constant envelope.

GMSK uses Gaussian pulse shaping, and has a complex envelope representation of

$$x(t)=\exp\left[jk_f\int_{-\infty}^{t} i(\tau)d\tau\right]=\exp\left[jk_f\int_{-\infty}^{t}\sum_{k=-\infty}^{\infty} a(kT_b)f(\tau-kT_b)d\tau\right] \qquad \textbf{(3.113)}$$

where $f(\tau - kT_b)$ is a Gaussian pulse function and $a(kT_b)$ is a unit-amplitude data source as defined in **Eq. (3.109)**. Since the Fourier transform of a Gaussian pulse in the time domain is a Gaussian pulse, it is seen that this

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modulation will cxhibit intrinsic band limiting, in contrast to PSK. In **Figure 10** the GMSK constellation diagram is illustrated, where it is seen that the envelope is constant. The information is contained in how rapidly the phase function moves from one location on the circle to another, in a fashion similar to FM. GMSK is used in the Global Standard for Mobile Communications (GSM) wireless system[3].

Probabilistic Envelope Characterization

The complex trajectory of a signal, determined by the modulation technique, band limiting, and signal coding used, is a time parametric representation of the instantaneous envelope of the signal. As such, the duration of a particular envelope level in combination with the transfer characteristics of the power amplifier (PA) will establish the resultant instantaneous saturation level. If the average spectral regrowth exhibited by a PA is considered a summation of many instantaneous saturation events, it follows that the more often an envelope induces saturation, the higher the average spectral regrowth will be. It is for this reason that the peak-to-average ratio, though widely used, is ill suited for estimating and comparing the linearity requirements of a PA[16].

This section introduces a method for probabilistically evaluating the time domain characteristics of a an arbitrary signal to establish its associated peak-to-average ratio and instantaneous envelope power distribution. The envelope distribution function (EDF) is introduced to estimate the peak power capability required of the PA and compare different signals. The EDF for many of the wireless systems presently in use are examined.

The Envelope Distribution Function

Let $p(t)$ be the instantaneous power of a signal $x(t)$, with an associated instantaneous power probability

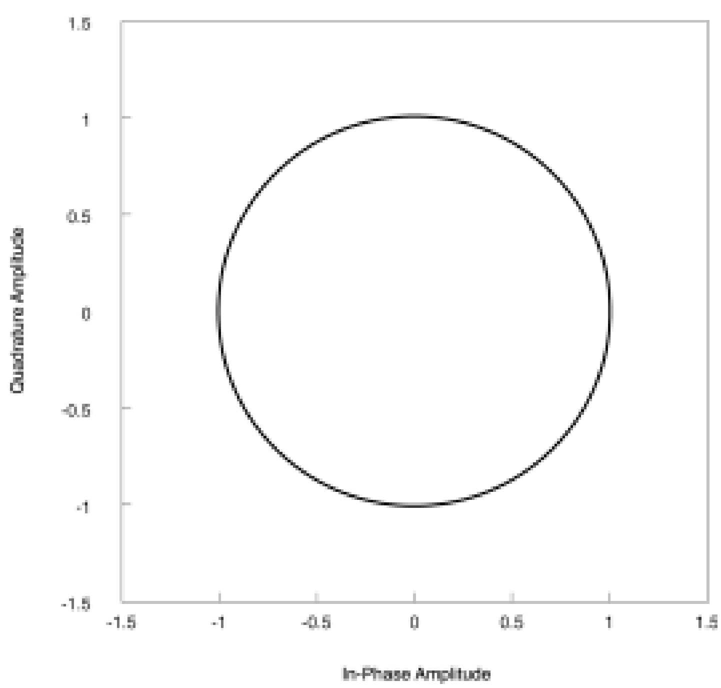

Figure 10. Constellation Diagram for GMSK, Which is Used in the GSM Wireless System.

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distribution function φ(p). The probability of the instantaneous power exceeding the mean signal power is

$$\Pr\left[\text{instantaneous power} > \text{mean power}\right] = \Psi\left(\bar{p}\right) = 1 - \int_{E[\bar{p}]}^{\infty} \varphi\left(\bar{p}\right) d\bar{p} \quad \textbf{(3.114)}$$

where Ep is the average power of *p(t)*. This function is defined as the envelope distribution function (EDF). In practice, the EDF is evaluated numerically, although it is possible to generate closed-form expressions. A specified probability of the EDF, typically 10^{-6}, is referred to as the peak-to-average ratio 6. This is defined as

$$\sigma = \frac{\text{EDF @ } 10^{-6}}{E\left[\tilde{p}(t)\right]} \quad \textbf{(3.115)}$$

A gradual roll off of the EDF indicates the instantaneous power is less likely to be near the mean power of the signal. This characteristic implies enhanced linearity performance to minimize increased distortion associated with the relative increase in the instantaneous clipping of the signal. Alternatively, for a given linearity requirement, the average power of the signal must necessarily decrease, resulting in lower effficiency. The amount that the average power must be reduced is referred to as output back off, and is usually specified in dB normalized to the 1 dB compression point of the PA under single-tone excitation. Finally, note that the EDF only has meaning for signals with a time-varying envelope.

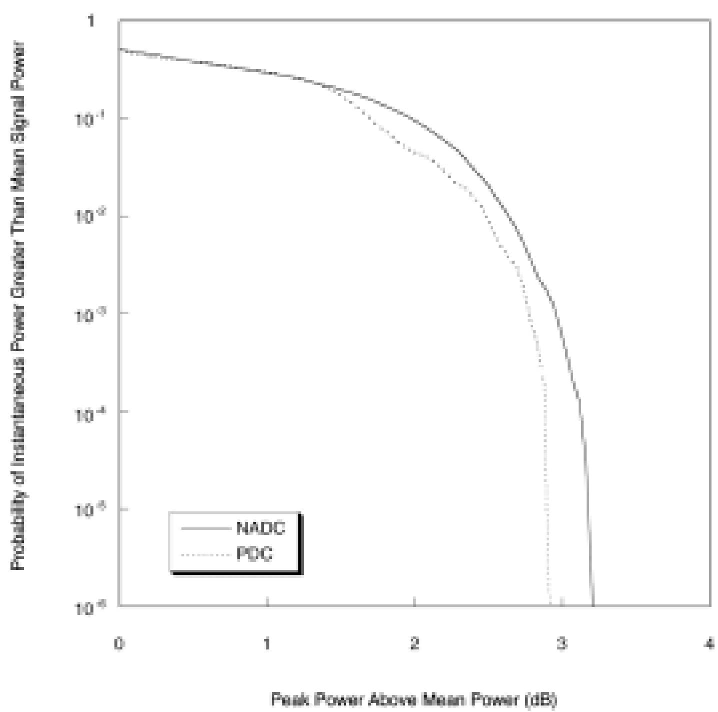

Figure 11. The EDF for NADC and PDC. NADC uses π/4-DQPSK with a = 0.35 and PDC uses π/4-DQPSK with a = 0.50. Note PDC has a Slightly Lower Peak-to-average Ratio due to the more Gradual Roll Off of the Filter Response.

The EDF for Various Wireless Systems

In this section, **Eq. (3.114)** is used to examine the EDF for several of the wireless systems described earlier. NADC, PDC, and ED&E are considered first. The EDF for CDMA-FL and CDMA-RL using only a pilot tone is considered next. The impact of traffic channel, synchronization channel, and paging channels on the CDMA-FL EDF is then illustrated, where it will be seen that a significant increase in the peak-toaverage ratio results, making this signal difficult to amplify. For comparison purposes, the EDF for a two-tone signal and complex Gaussian noise will also be shown. The EDF for each of these signals is shown in **Figures 11** through **16**.

Summary

Contemporary microwave circuit design requires a basic understanding of digital modulation theory in order to meet the needs of a customer who ultimately speaks in terms of communication theory. This chapter was intended to provide a brief overview of the signal analysis tools necessary for the microwave engineer to understand digital modulation and how it impacts the design and characterization of microwave circuits used in contemporary wireless communication systems.

Complex envelope analysis was introduced as a means to describe arbitrarily modulated signals, leading to a geometric interpretation of modulation. The necessity and subsequent implications of band-limiting PSK signals were discussed. As an alternative to PSK, CPM was also introduced.

Signal analysis methods are often used to simplify the design process. Although the peak-to-average ratio of a signal is widely used to estimate of the linearity requirements of a PA, it was shown that this metric is ill suited in general for this purpose due to the random distribution of instantaneous power of a signal. The envelope distribution function (EDF) was introduced as means to compare various signals and to provide a more accurate estimate of the required linearity performance of a PA.

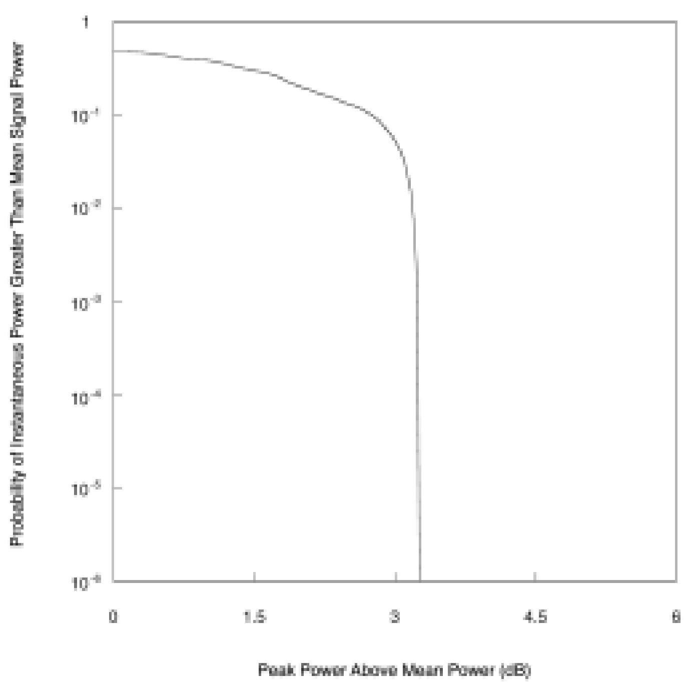

Figure 12. The EDF for EDGE (With all Time Slots Active). Compare to Figure 15, Where the EDF for a Two-tone Signal is Shown.

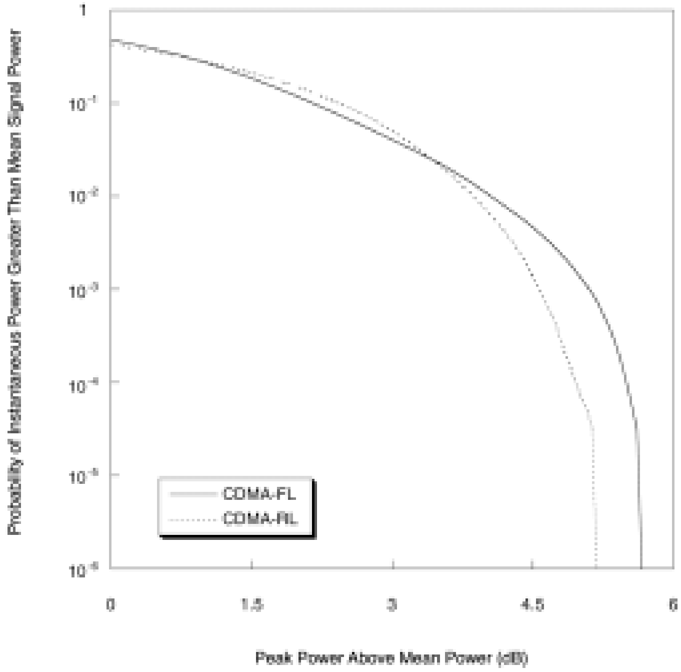

Figure 13. The EDF for Forward-link CDMA and Reverse-link CDMA with Pilot Tone Only. Note that Although CDMA-RL has a Lower Peak-to-average Ratio, in Areas of High Probability of Occurrence it has a Higher Peak Power and Results in Higher Spectral Regrowth than CDMA-FL. This Comparison Clearly Illustrates the Advantages of Using the EDF over the Peak-to-average Ratio.

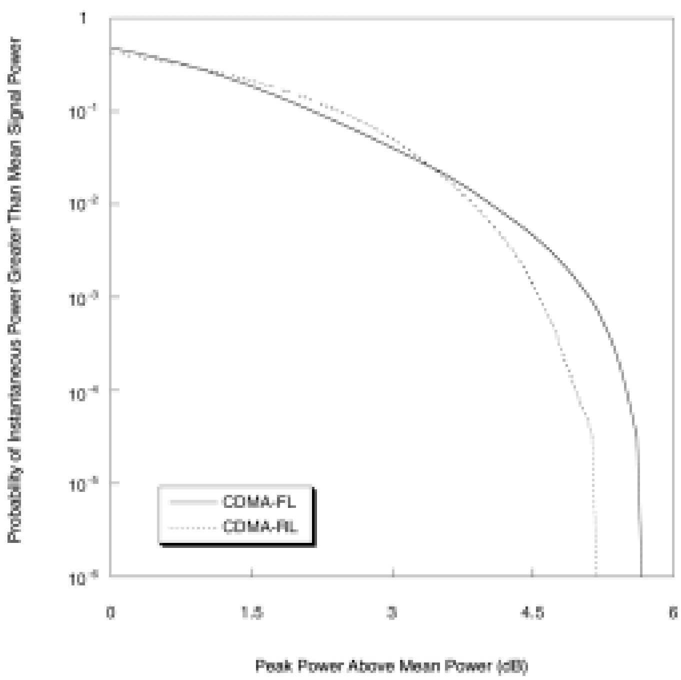

Figure 14. The EDF for Forward-link CDMA with Six Traffic Channels Active and Synchronization, Paging, and Pilot also Active. Compare to Figure 13 and Observe the Significant Increase in Peak-to-average Ratio.

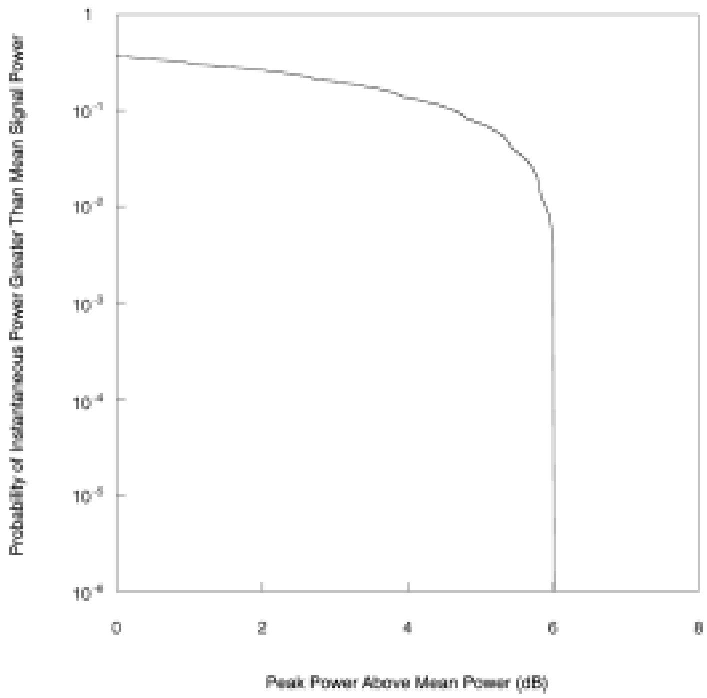

Figure 15. The EDF for a Two-tone Signal. Note that Like the EDGE Signal, the Two-tone Signal Exhibits a Gradual Roll Off, Leading to Increased Distortion with Respect to a Similar Signal with the Same Peak-to-average Ratio but a Faster Roll Off, Such as CDMA-FL.

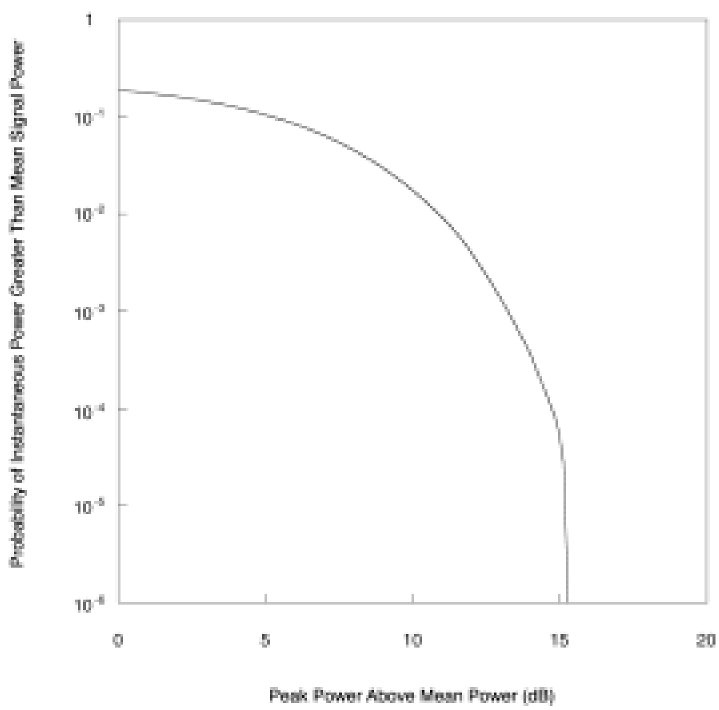

Figure 16. The EDF for a Complex White Gaussian Noise Signal (Commonly used for Noise-to-power Ratio Characterization). The Theoretical Peak-to-average of this Signal is Infinite, but in Practice is Approximately 10 dB to 15 dB, Depending on the Length the Sequence Used.

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References

1 W. C. Y. Lee, Mobile Cellular Telecommunications Systems, New York: McGraw-Hill, 1989.

2 ELMTIA Interim Standard, Recommended Minimum Performance Standards for 800 MHz Dual Mode Mobile Stations, EIA/TIA/IS-55, 1991.

3 M. R. L. Hodges, The GSM Radio Interface, British Telecom. Tech. Journal, 8, 31-43, January 1990.

4 EIA/TLA Interim Standard, Mobile Station—Base Station Compatibility Standardfor Dual-Mode Wideband Spread Spectrum Cellular System, TIA/EL\/IS-95, 1993

5 R. N. Bracewell, The Fourier Transform and Its Application, New York: McGraw-Hill, 1978.

6 L. W. Couch, Digital and Analog Communications, 4th ed. New York: Macmillan, 1994.

7 M. Jeruchim, R Balaban, and K. Shanmugan, Simulation of Communication Systems, New York: Plenum Press, 1992.

8 A. Papuolis, Probability, Random Variables, and Stochastic Processes, 3rd ed., New York McGraw Hill, 1991.

9 M. K. Simon, S. M. Hinedi, and W. C. Lindsey, Digital Communication Techniques: Signal Design and Detection, New York: Prentice-Hall, 1994.

10 M. Schwartz, Information Transmission, Modulation, and No*e, 3rd ed., New York: McGraw-Hill, 1980.

11 W. R. Bennett and J. R. Davey, Data Transmission, New York: McGraw-Hill, 1965.

12 H. Nyquist, Certain topics in telegraph transmission theory, Transactions of the AIEE, 47, 117- 141, February 1928.

13 J. G. Proakis, Digital Communications, 2nd ed., New York: McGraw-Hill, 1992.

14 R. Schaefer and A. Openheim, Discrete Time Signal Processing, New York: McGraw-Hill, 1992.

15 Research and Development Center for Radio Systems (RCR), Digital Cellular Communicanon Systems, RCR STD-27, April 1991.

16 J. E. Sevic and M. B. Steer, On the Significance of Envelope Peak-to-Average Ratio for Estimating the Spectral Regrowth Characteristics of an RF/Microwave Power Amplifier, IEEE Transactions on Microwave Theory and Techniques, in press.

17 ETSI Standard GSM 11.21: Digital Cellular Telecommunications System BSS Radio Aspects, ETSI, 1999.